



KING SAUD UNIVERSITY  
*College of Science*  
*Department of Mathematics*

# M-106

Second Semester (1431/1432)

Solution First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20

Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

## Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	$a$	$d$	$a$	$d$	$b$	$c$	$a$	$a$	$c$	$c$

Q. No: 1 If  $\sum_{k=1}^5 (\alpha k^2 + k - 1) = 20$ , then the value of  $\alpha$  is equal to:

- (a)  $\frac{2}{11}$                       (b)  $\frac{-2}{11}$                       (c)  $\frac{1}{11}$                       (d)  $\frac{-1}{11}$

Q. No: 2 The value of the integral  $\int_1^e \frac{e^{\ln(x^3)}}{x} dx$  is equal to:

- (a)  $\frac{1}{2}e^2 - \frac{1}{2}$                       (b)  $e^3 - 1$                       (c)  $\frac{1}{3}e^3$                       (d)  $\frac{1}{3}e^3 - \frac{1}{3}$

Q. No: 3 The number  $z$  that satisfies the conclusion of the Mean value Theorem for  $f(x) = 1 + x^2$  on  $[-3, 0]$  is:

- (a)  $-\sqrt{3}$                       (b)  $\sqrt{3}$                       (c)  $\sqrt{2}$                       (d)  $-\sqrt{2}$

Q. No: 4 The average value of  $f(x) = |x - 1|$  on  $[0, 1]$  is equal to:

- (a)  $-\frac{1}{2}$                       (b)  $\frac{3}{2}$                       (c)  $0$                       (d)  $\frac{1}{2}$

Q. No: 5 If  $F(x) = \int_1^{x^3} \sqrt{5 + t^2} dt$ , then  $F'(1)$  is equal to:

- (a)  $0$                       (b)  $3\sqrt{6}$                       (c)  $\sqrt{6}$                       (d)  $\frac{2}{\sqrt{6}}$

Q. No: 6 The derivative of the function  $f(x) = \tan^{-1}(\sinh x)$  is equal to:

- (a)  $\frac{1}{1 + \sinh^2 x}$                       (b)  $\sec^2(\sinh x)$                       (c)  $\frac{1}{\cosh x}$                       (d)  $\frac{\cosh x}{1 - \sinh^2 x}$

Q. No: 7 The value of the integral  $\int_0^1 (7x)7^{x^2} dx$  is equal to:

- (a)  $\frac{21}{\ln 7}$                       (b)  $21 \ln 7$                       (c)  $\frac{49}{\ln 7}$                       (d)  $\frac{7}{\ln 7}$

Q. No: 8 If  $F(x) = x^{\frac{1}{x}}$ , then  $F'(e)$  is equal to:

- (a)  $0$                       (b)  $e^{\frac{1}{e}}$                       (c)  $e$                       (d)  $\frac{1}{e^2}$

Q. No: 9 The value of the integral  $\int \frac{\cos x}{\sqrt{4 + \sin x}} dx$  is equal to:

- (a)  $\frac{1}{2}\sqrt{\sin x + 4} + c$                       (b)  $\sqrt{\sin x + 4} + c$                       (c)  $2\sqrt{\sin x + 4} + c$                       (d)  $-2\sqrt{\sin x + 4} + c$

Q. No: 10 The value of the integral  $\int \frac{\sinh x}{9 + \cosh^2 x} dx$  is equal to:

- (a)  $\tan\left(\frac{\cosh x}{3}\right) + c$  (b)  $\ln(9 + \cosh^2 x) + c$  (c)  $\frac{1}{3} \tan^{-1}\left(\frac{\cosh x}{3}\right) + c$  (d)  $\tan^{-1}\left(\frac{\cosh x}{3}\right) + c$

## Full Questions

Question No: 11 Approximate the integral  $\int_0^2 \frac{1}{1 + \sqrt[3]{x}} dx$  using the **Trapezoidal rule**

with  $n = 4$ . [3].

**Solution:**

$$\text{Let } f(x) = \frac{1}{1 + \sqrt[3]{x}}.$$

$$\Delta x = \frac{2}{4} = 0.5$$

$$x_0 = 0, \quad x_1 = 0.5, \quad x_2 = 1, \quad x_3 = 1.5 \quad \text{and} \quad x_4 = 2. \quad (1)$$

$$\begin{aligned} \int_0^2 \frac{1}{1 + \sqrt[3]{x}} dx &\approx \frac{2-0}{2 \times 4} \{f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + f(2)\} & (1) \\ &= \frac{1}{4} \{1 + 2(0.55751) + 2(0.5) + 2(0.46626) + 0.44249\} \\ &= \frac{1}{4} \{1 + 1.115 + 1 + 0.93252 + 0.44249\} \\ &= \frac{1}{4} \{4.49\} \approx 1.1225 & (1) \end{aligned}$$

Question No: 12 If  $y(x) = (x^3 + 1)^2 (x^2 + 1)^3 (x^4 + 1)^5$  then find  $y'(x)$ . [2]

**Solution:**

$$\ln y = 2 \ln(x^3 + 1) + 3 \ln(x^2 + 1) + 5 \ln(x^4 + 1) \quad (1)$$

$$\frac{y'}{y} = \frac{6x^2}{x^3+1} + \frac{6x}{x^2+1} + \frac{20x^3}{x^4+1}$$

$$\text{So } y' = \left[ \frac{6x^2}{x^3+1} + \frac{6x}{x^2+1} + \frac{20x^3}{x^4+1} \right] (x^3 + 1)^2 (x^2 + 1)^3 (x^4 + 1)^5 \quad (1).$$

Question No: 13 Evaluate the integral  $\int \frac{x+1}{x\sqrt{25-x^2}} dx$ . [2]

**Solution:**

$$\begin{aligned} \int \frac{x+1}{x\sqrt{25-x^2}} dx &= \int \frac{1}{\sqrt{25-x^2}} dx + \int \frac{1}{x\sqrt{25-x^2}} dx \\ &= \sin^{-1}\left(\frac{x}{5}\right) - \frac{1}{5} \operatorname{sech}^{-1}\left(\frac{|x|}{5}\right) + c \end{aligned} \quad (1+1)$$

Question No: 14 Evaluate the integral  $\int \frac{1}{x\sqrt{x^8-16}} dx$ . [3]

**Solution:**

$$\text{Let } u = x^4 \Rightarrow du = 4x^3 dx \quad (1)$$

$$\begin{aligned} \int \frac{1}{x\sqrt{x^8-16}} dx &= \frac{1}{4} \int \frac{1}{u\sqrt{u^2-16}} du \quad (1) \\ &= \frac{1}{16} \sec^{-1}\left(\frac{u}{4}\right) + c \\ &= \frac{1}{16} \sec^{-1}\left(\frac{x^4}{4}\right) + c \quad (1) \end{aligned}$$