KING SAUD UNIVERSITY
*College of Science*
*Department of Mathematics*

# M-106

## Solution Second Semester (1432/1433)

## M-106
## Second Mid-Exam

| Name: | Number: |
|---|---|
| Name of Teacher: | Group No: |

Max Marks: 20 Time: 90 minutes

Marks:

| | |
|---|---|
| Multiple Choice (1-10) | |
| Question # 11 | |
| Question # 12 | |
| Question # 13 | |
| Question # 14 | |
| Total | |

# Multiple Choice

| $Q.\ No:$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\{a,b,c,d\}$ | $b$ | $b$ | $c$ | $a$ | $a$ | $a$ | $c$ | $b$ | $d$ | $a$ |

Q. No: 1 $\lim_{x\to 0}\dfrac{2^x-3^x}{x}$ is equal to:

$(a)\ \ \infty$ $\qquad (b)\ \ln\dfrac{2}{3}$ $\qquad (c)\ \ \ln\dfrac{3}{2}$ $\qquad (d)\ \ -1$

Q. No: 2 The partial fraction decomposition of $\dfrac{x^2+2}{(x^4-1)(x-1)}$ takes the form:

$(a)\ \dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{x^2-1}+\dfrac{E}{x-1}$ $\quad (b)\ \dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-1}+\dfrac{D}{(x-1)^2}+\dfrac{E}{x+1}$

$(c)\ \dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{(x-1)^2}+\dfrac{E}{x+1}$ $\quad (d)\ \dfrac{Ax+B}{x^2+1}+\dfrac{C}{(x-1)^2}+\dfrac{E}{x+1}$

Q. No: 3 The integral $\displaystyle\int \frac{2dx}{x^2-4x+3}$ is equal to:

$(a)\ \ln(x^2-4x+3)+c$ $\quad (b)\ \ln(\left|\dfrac{x+1}{x+3}\right|)+c$

$(c)\ \ln(\left|\dfrac{x-3}{x-1}\right|)+c$ $\quad (d)\ \ln(\left|\dfrac{x-1}{x-3}\right|)+c$

Q. No: 4 The value of the integral $\displaystyle\int \sin^5(x)\cos^3(x)\,dx$ is equal to:

$(a)\ \ \tfrac{1}{6}\sin^6(x)-\ \tfrac{1}{8}\sin^8(x)\ +c$ $\qquad (b)\ \ \tfrac{1}{5}\sin^5(x)-\ \tfrac{1}{3}\sin^3(x)\ +c$

$(c)\ \ \tfrac{1}{3}\sin^5(x)-\ \tfrac{1}{2}\sin^2(x)\ +c$ $\qquad (d)\ \ \tfrac{1}{3}\sin^5(x)-\ \tfrac{1}{8}\sin^8(x)\ +c$

Q. No: 5 The substitution $u=\tan\left(\frac{x}{2}\right)$ transforms the integral $\displaystyle\int \frac{1}{3-\sin x+2\cos x}dx$ into:

$(a)\ \displaystyle\int \frac{2}{u^2-2u+5}du$ $\quad (b)\ \displaystyle\int \frac{2}{u^2-2u+3}du$

$(c)\ \displaystyle\int \frac{2}{u^2+2u+5}du$ $\quad (d)\ \displaystyle\int \frac{2}{u^2+2u+3}du$

Q. No: 6 To evaluate the integral $\int \frac{dx}{x^2\sqrt{x^2-25}}$, we use the substitution:

$(a)$ $x = 5\sec(\theta)$ $(b)$ $x = \sec^5(\theta)$ $(c)$ $x = 5\tan(\theta)$ $(d)$ $x = \tan^5(\theta)$

Q. No: 7 The improper integral $\int_0^{+\infty} \frac{dx}{x^2+1}$

$(a)$ converges to $\pi$ $(b)$ diverges $(c)$ converges to $\frac{\pi}{2}$ $(d)$ converges to $+\infty$

Q. No: 8 The value of the integral $\int \frac{1}{1+e^x}dx$ is equal to:

$(a)$ $x - \ln(x+1) + c$ $(b)$ $x - \ln(e^x+1) + c$

$(c)$ $\frac{x^2}{2} - \ln(e^x+1) + c$ $(d)$ $\ln(\frac{x^2}{2}) - \ln(x+1) + c$

Q. No: 9 The value of the integral $\int \ln(x)dx$ is equal to:

$(a)$ $\frac{\ln(x)}{x} - x + c$ $(b)$ $x\ln(x) - \ln(x) + c$ $(c)$ $\frac{\ln^2(x)}{2}$ $(d)$ $x\ln(x) - x + c$

Q. No: 10 The area of the region bounded by the graphs of $y = x$, $y = -x$ and $y = 1$ is equal to:

$(a)$ 1 $(b)$ 0 $(c)$ 2 $(d)$ $\frac{1}{2}$

# Full Questions

Question No: 11 **Evaluate** $\int \sin^5(5x)\,dx$

**Solution:** Put

$$u = 5x \text{ then } du = 5dx \quad [0.5]$$

and we will have

$$\begin{aligned}\int \sin^5(5x)\,dx &= \frac{1}{5}\int \sin^5(u)\,du = \frac{1}{5}\int \sin^4(u)\sin(u)du = \frac{1}{5}\int (\sin^2(u))^2\sin(u)du \\ &= \frac{1}{5}\int (1-\cos^2(u))^2\sin(u)du \quad [0.5]\end{aligned}$$

let now consider

$$v = \cos(u) \text{ then } dv = -\sin(u)du \quad [0.5]$$

and we will have

$$\int (1-\cos^2(u))^2\sin(u)du = \int (1-v^2)^2(-dv) = -\int (1-2v^2+v^4)du = -v+2\frac{v^3}{3}-\frac{v^5}{5}+c \quad [1]$$

Finally we will have

$$\begin{aligned}\int \sin^5(5x)\,dx &= -\frac{v}{5}+2\frac{v^3}{15}-\frac{v^5}{25}+\frac{c}{5} = -\frac{\cos(u)}{5}+2\frac{\cos^3(u)}{15}-\frac{\cos^5(u)}{25}+C \\ &= -\frac{\cos(5x)}{5}+2\frac{\cos^3(5x)}{15}-\frac{\cos^5(5x)}{25}+C \quad [0.5]\end{aligned}$$

Question No: 12 **Evaluate** $\int \frac{x+1}{x^2+x-2}dx$

**Solution:** We have

$$\frac{x+1}{x^2+x-2} = \frac{2/3}{x-1}+\frac{1/3}{x+2} \quad [1]$$

and then

$$\int \frac{x+1}{x^2+x-2}dx = \frac{2}{3}\ln(x-1)+\frac{1}{3}\ln(x+2)+c \ [1]$$

Question No: 13 **Evaluate** $\int \tan(x)\sec^3(x)dx$ [**3**]

**Solution:** Put

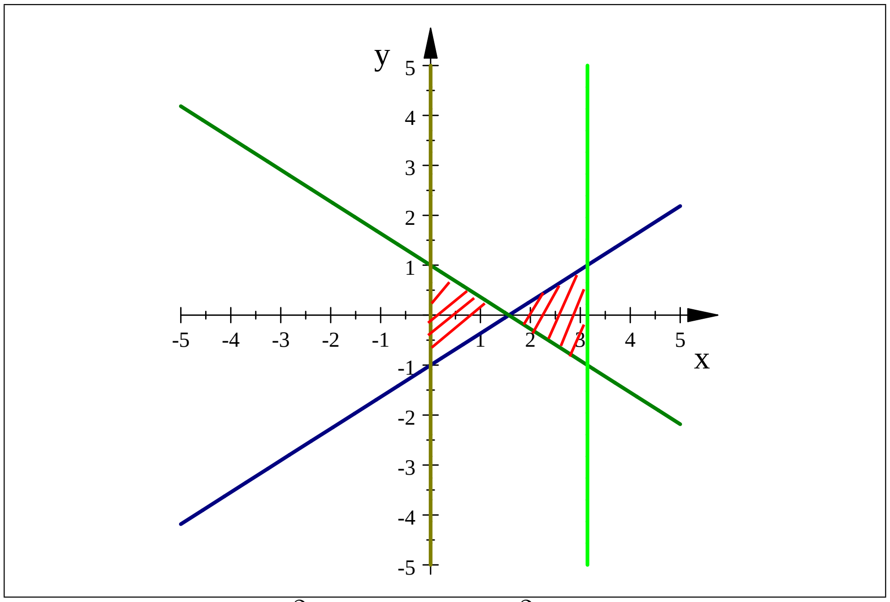
$$u = \sec(x\ ) \text{ then } du = \sec(x)\tan(x)dx \quad [1]$$

and we have

$$\begin{aligned}\int \tan(x)\sec^3(x)dx &= \int \sec^2(x)\sec(x)\tan(x)dx = \int u^2 du \ [1] \\ &= \frac{u^3}{3} + c = \frac{\sec^3(x)}{3} + c \quad [1]\end{aligned}$$

Question No: 14 **Sketch** the region bounded by $y = -\frac{2}{\pi}x + 1$ and $y = \frac{2}{\pi}x - 1$, $x = 0$ and $x = \pi$. Find it's **area**.

**Solution:**




$$y = -\frac{2}{\pi}x + 1 \text{ and } y = \frac{2}{\pi}x - 1 \quad [1]$$

$$\begin{aligned}A &= \int_0^{\frac{\pi}{2}} ((-\frac{2}{\pi}x + 1) - (\frac{2}{\pi}x - 1))dx + \int_{\frac{\pi}{2}}^{\pi} ((\frac{2}{\pi}x - 1) - (-\frac{2}{\pi}x + 1))dx \\ &= \int_0^{\frac{\pi}{2}} (-\frac{4}{\pi}x + 2)dx + \int_{\frac{\pi}{2}}^{\pi} (\frac{4}{\pi}x - 2)dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi \quad [1]\end{aligned}$$