KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

Second Semester (1430/1431)

Solution Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	c	a	a	d	b	b	d	a	b	d

Q. No: 1 $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$ is equal to:

(a) 1 (b) e (c) e^2 (d) ∞

Q. No: 2 The partial fraction decomposition of $\dfrac{1}{x^3+1}$ takes the form:

(a) $\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$ (b) $\frac{A}{x+1}+\frac{Bx+C}{x^2+x+1}$ (c) $\frac{A}{x+1}+\frac{Bx+C}{x^2+1}$ (d) $\frac{A}{x+1}+\frac{C}{x^2-x+1}$

Q. No: 3 To evaluate the integral $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}dx$, we use the substitution:

(a) $x = 2\tan\theta$ (b) $\theta = 2\tan x$ (c) $x = 2\sec\theta$ (d) $x = 2\sin\theta$

Q. No: 4 The value of the integral $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4(x)\cos(x)\,dx$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{1}{5}\left(\frac{\pi}{2}\right)^5$ (c) $\left(\frac{\pi}{2}\right)^5$ (d) $\frac{1}{5}$

Q. No: 5 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\displaystyle\int \frac{1}{1+2\sin x}dx$ into:

(a) $\int \frac{1}{u^2+4u+1}du$ (b) $\int \frac{2}{u^2+4u+1}du$ (c) $\int \frac{2}{u^2-4u+1}du$ (d) $\int \frac{1}{1+2u}du$

Q. No: 6 To evaluate the integral $\displaystyle\int \frac{\sec x}{\cot^3(x)}dx$, we use the substitution:

(a) $u = \tan x$ (b) $u = \sec x$ (c) $u = \cot x$ (d) $u = \sin x$

Q. No: 7 The improper integral $\displaystyle\int_0^{-1} \frac{1}{\sqrt[3]{x+1}}dx$

(a) converges to $\frac{1}{2}$ (b) diverges (c) converges to $\frac{3}{2}$ (d) converges to $\frac{-3}{2}$

Q. No: 8 The value of the integral $\displaystyle\int \frac{1}{\sqrt{x^2-8x+25}}dx$ is equal to:

(a) $\sinh^{-1}\left(\frac{x-4}{3}\right)+c$ (b) $\sinh^{-1}(x-4)+c$ (c) $\sin^{-1}\left(\frac{x-4}{3}\right)+c$
(d) $\frac{1}{3}\sinh^{-1}\left(\frac{x-4}{3}\right)+c$

Q. No: 9 The improper integral $\int_{e}^{\infty} \frac{1}{x\left(\ln x\right)^{2}} dx$

(a) converges to 0 (b) converges to 1 (c) diverges (d) converges to -1

Q. No: 10 The area of the region bounded by the graphs of $x = y^2$ and $x = 2 - y^2$ is equal to:

(a) $\frac{1}{3}$ (b) 8 (c) 1 (d) $\frac{8}{3}$

Full Questions

Question No: 11 **Evaluate** $\int x \sec^2(x)\, dx$ [**2**]

Solution:

Let $\begin{cases} u = x \\ v' = \sec^2 x \end{cases} \Rightarrow \begin{cases} u' = 1 \\ v = \tan x \end{cases}$

So

$$\begin{aligned} \int x \sec^2(x)\, dx &= x \tan x - \int \tan x dx \\ &= x \tan x + \ln\left|\cos x\right| + c. \end{aligned}$$

Question No: 12 **Evaluate** $\int \frac{1}{x^3 - x^2} dx$ [**3**]

Solution:

$$\begin{aligned} \frac{1}{x^3 - x^2} &= \frac{1}{x^2(x-1)} \\ &= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} \\ &= \frac{(-1)}{x} + \frac{(-1)}{x^2} + \frac{1}{(x-1)}. \end{aligned}$$

So

$$\begin{aligned} \int \frac{1}{x^3 - x^2} dx &= \int \left[\frac{(-1)}{x} + \frac{(-1)}{x^2} + \frac{1}{(x-1)}\right] dx \\ &= -\ln|x| + \frac{1}{x} + \ln|x-1| + c. \end{aligned}$$

Question No: 13 **Evaluate** $\int \frac{1}{x^2\sqrt{x^2-1}}dx$ [**3**]

Solution:

Let $x = \sec\theta$, where θ in $\left(0, \frac{\pi}{2}\right)$ or in $\left(\pi, \frac{3\pi}{2}\right)$. $(dx = \sec\theta\tan\theta d\theta)$

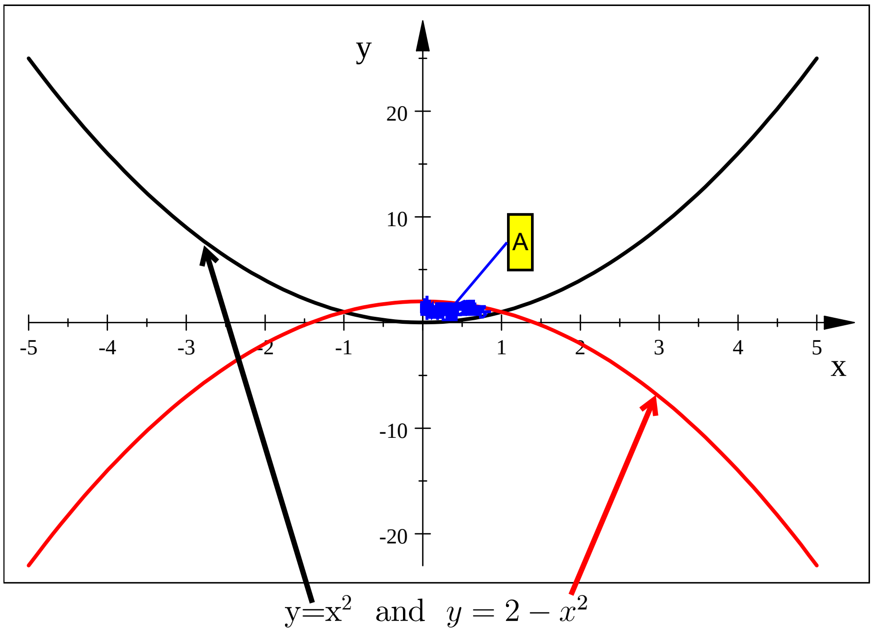
In this case $\tan\theta \geq 0$ and $\sqrt{x^2-1} = \sqrt{(\sec\theta)^2 - 1} = \sqrt{(\tan\theta)^2} = \tan\theta$.

So

$$\begin{aligned}\int \frac{1}{x^2\sqrt{x^2-1}}dx &= \int \frac{\sec\theta\tan\theta}{(\sec\theta)^2\tan\theta}d\theta \\ &= \int \frac{1}{\sec\theta}d\theta = \int \cos\theta d\theta \\ &= \sin\theta + c = \frac{\sqrt{x^2-1}}{x} + c\end{aligned}$$

Question No: 14 Let R be the region bounded by the graphs of $y = x^2$ and $y = 2 - x^2$ for $0 \leq x \leq 1$. **Sketch** the region R and **set up** (Do not evaluate) an integral that can be used to find its **area**. [**2**]

Solution:



y=x^2 and $y = 2 - x^2$

So $A = \int_0^1 (2 - x^2 - x^2)\, dx$