KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1432/1433)

Solution Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	c	a	c	d	b	d	c	b	c	d

Q. No: 1 $\lim_{x\to 0^+} \dfrac{\ln x}{\ln(\sin x)}$ is equal to:

(a) ∞ (b) 0 (c) 1 (d) $-\infty$

Q. No: 2 If $\dfrac{1}{(x-4)(x+2)} = \dfrac{A}{(x-4)} + \dfrac{B}{(x+2)}$, then the value of A is equal to:

(a) $\frac{1}{6}$ (b) 4 (c) $\frac{1}{4}$ (d) $\frac{-1}{6}$

Q. No: 3 To evaluate the integral $\int \sqrt{4x^2-16}dx$, we use the substitution:

(a) $x = 4\sec\theta$ (b) $x = 2\cos\theta$ (c) $x = 2\sec\theta$ (d) $x = 2\tan\theta$

Q. No: 4 The value of the integral $\int_0^{\frac{\pi}{2}} \cos^5(x)\sin(x)\,dx$ is equal to:

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) $\frac{1}{6}$

Q. No: 5 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\int \dfrac{1}{1+\cos x}dx$ into:

(a) $\int 2du$ (b) $\int du$ (c) $\int \dfrac{1}{1+u}du$ (d) $\int \dfrac{1}{1+u^2}du$

Q. No: 6 If $\int (\sec x)^{\frac{3}{2}} \tan x dx = \int \sqrt{u}\,du$ then

(a) $u = \tan x$ (b) $u = (\sec x)^{\frac{3}{2}}$ (c) $u = \sqrt{\sec x}$ (d) $u = \sec x$

Q. No: 7 The improper integral $\int_0^{\infty} \dfrac{1}{x^2+4}dx$

(a) converges to 0 (b) diverges (c) converges to $\frac{\pi}{4}$ (d) converges to $\frac{\pi}{2}$

Q. No: 8 The value of the integral $\int \dfrac{1}{\sqrt{4x-x^2}}dx$ is equal to:

(a) $\sinh^{-1}\left(\frac{x-2}{2}\right)+c$ (b) $\sin^{-1}\left(\frac{x-2}{2}\right)+c$ (c) $\frac{1}{2}\sin^{-1}\left(\frac{x-2}{2}\right)+c$
(d) $\sin^{-1}\left(\frac{x+2}{2}\right)+c$

Q. No: 9 The area of the region **bounded** by the graphs of equations: $y = x^2$ and $y = -x$ is equal to:

$(a)\ \frac{5}{6}$ $(b)\ 2$ $(c)\ \frac{1}{6}$ $(d)\ \frac{1}{3}$

Q. No: 10 The value of the integral $\int \tan^3(x)\sec(x)\,dx$ is equal to:

$(a)\ \frac{1}{3}\sec^3 x + \sec x + c$ $(b)\ \frac{-1}{3}\sec^3 x - \sec x + c$

$(c)\ \frac{-1}{3}\sec^3 x + \sec x + c$ $(d)\ \frac{1}{3}\sec^3 x - \sec x + c$

Full Questions

Question No: 11 **Evaluate** $\int 2x\tan^{-1}(x)\,dx$ [**2**]

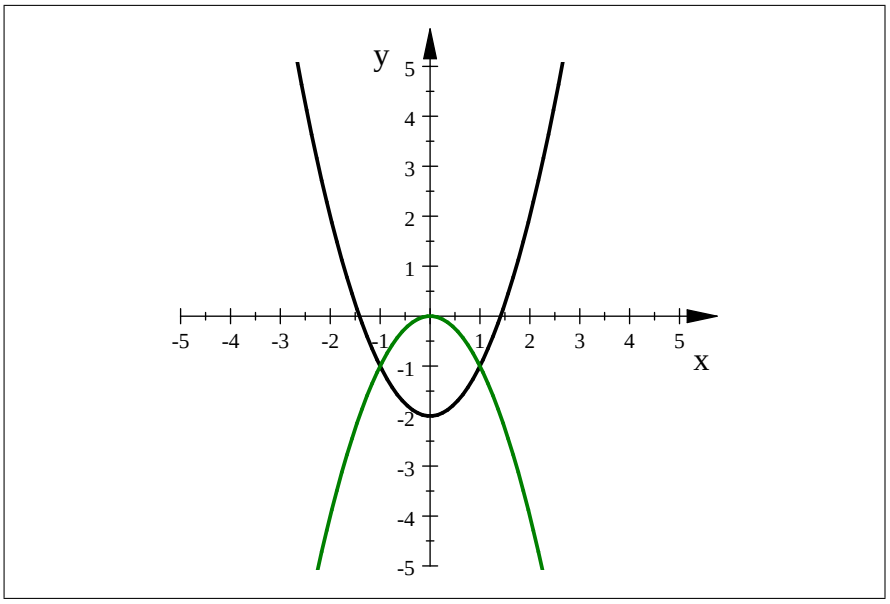
Solution: Let $\begin{cases} u = \tan^{-1} x \\ v' = 2x \end{cases}$, then $\begin{cases} u' = \frac{1}{1+x^2} \\ v = x^2 \end{cases}$

So

$$\begin{aligned} \int 2x\tan^{-1}(x)\,dx &= x^2\tan^{-1} x - \int \tfrac{x^2}{1+x^2}\,dx && (0.5) \\ &= x^2\tan^{-1} x - x + \tan^{-1} x + c && (0.5+0.5+0.5) \end{aligned}$$

Question No: 12 **Sketch** the region R **bounded** by the graphs of $y = x^2 - 2$; $y = -x^2$. and **find** its area. [**2**]

Solution: Graph (1)



$$y = x^2 - 2 \quad \text{and} \quad y = -x^2$$

$$A = \int_{-1}^{1} (-x^2 - x^2 + 2)\, dx = \tfrac{8}{3}. \qquad (0.5 + 0.5)$$

Question No: 13 **Evaluate** $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}dx$ [**3**]

Let $x = 2\sin\theta, \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}. \quad (dx = 2\cos\theta d\theta)$ (0.5)

$\sqrt{4-x^2} = \sqrt{4 - 4\sin^2\theta} = 2\cos\theta.$

So

$$\begin{aligned}\int \frac{1}{x^2\sqrt{4-x^2}}dx &= \tfrac{1}{4}\int \frac{1}{\sin^2\theta}d\theta && (0.5)\\ &= -\tfrac{1}{4}\cot\theta + c && (1)\\ &= -\tfrac{1}{4}\frac{\sqrt{4-x^2}}{x} + c && (1)\end{aligned}$$

Question No: 14 **Evaluate** $\displaystyle\int \frac{x-1}{x^2+x}dx$ [**3**]

Solution:

$$\frac{x-1}{x^2+x} = -\frac{1}{x} + \frac{2}{x+1} \qquad (0.5 + 0.5)$$

So

$$\begin{aligned}\int \frac{x-1}{x^2+x}dx &= \int -\frac{1}{x}dx + \int \tfrac{2}{x+1}dx\\ &= -\ln|x| + 2\ln|x+1| + c \qquad (1+1)\end{aligned}$$