King Saud University: Mathematics Department MaTH-254
Summer Semester 1439-40 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:________________ I.D. No. ________________

Name of the Teacher:________________ Section No. ________________

Note: Check the total number of pages are Five (5).

(15 Multiple choice questions and Two (2) Full questions)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one $(2 \times 15 = 30)$

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d															

Quest. No.	Marks
Q. 1 to Q. 15	
Q. 16	
Q. 17	
Total	

Question 1: The number of bisections required to solve the equation $x^3 + x = 1$ in $[0, 1]$ accurate to within 10^{-3} is:

(a) 10 (b) 8 (c) 9 (d) 11

Question 2: Let $x^2 - e^x = 0$. Use Newton's Method and the initial approximation $x_0 = 0$ to find the first approximation is:

(a) $x_1 = 0$ (b) $x_1 = -1$ (c) $x_1 = 1$ (d) $x_1 = -2$

Question 3: The order of multiplicity of the root $\alpha = 1$ of the equation $x^4 - x^3 - 3x^2 + 5x - 2 = 0$ is:

(a) 2 (b) 1 (c) 4 (d) 3

Question 4: The next iterative value of the root of $x^2 - 4 = 0$ using secant method, if the initial guesses are 3 and 4, is :

(a) 2.5000 (b) 2.2857 (c) 5.5000 (d) 5.7143

Question 5: In the Gauss elimination with partial pivoting method for solving a system of linear algebraic equations, triangularzation leads to a matrix:

(a) Upper triangular (b) Lower triangular (c) Diagonal (d) Singular

Question 6: If $\hat{x} = [0.5, 0.0]^T$ is an approximate solution for the system $2x - y = 1$, $x + y = 2$, then the l_∞-norm of the corresponding residual vector is:

(a) 0.25 (b) 0.5 (c) 2.5 (d) 1.5

Question 7: The Lagrange polynomial that passes through the data points $(15, 24), (18, 37), (22, 25)$ is $p_2(x) = 24L_0(x) + 37L_1(x) + 25L_2(x)$. The value of $L_1(16)$ is:

(a) 0.071430 (b) 0.57143 (c) 0.5000 (d) 4.3333

Question 8: The Newtons divided difference second order polynomial for the data points $(15, 24), (18, 37), (22, 25)$ is $p_2(x) = b_0 + b_1(x - 15) + b_2(x - 15)(x - 18)$. The value of b_1 is:

(a) 1.0480 (b) 4.3333 (c) 0.14333 (d) 24.000

Question 9: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, if $\max_{0 \leq x \leq 0.3} f^{(5)}(x) = 1$, then the error bound in approximating $f(0.25)$ by using a fourth degree interpolating polynomial is bounded by:

(a) 0.78×10^{-5} (b) 0.78×10^{-8} (c) 0.78×10^{-6} (d) 0.78×10^{-9}

Question 10: When using the two-point forward formula with $h = 0.2$ for approximating the value of $f'(1)$, where $f(x) = \ln(x + 1)$, we have the computed approximation (accurate to 4 decimal places):

(a) 0.4666 (b) 0.4966 (c) 0.4766 (d) 0.4866

Question 11: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$, then the worst approximation of $f''(0.15)$ using 3-point difference formula is:

(a) 44.444 (b) −33.333 (c) 33.333 (d) −44.444

Question 12: The value of $\int_{0.2}^{2.2} xe^{x}\,dx$ by the using simple trapezoidal rule is most nearly is:

(a) 20.099 (b) 11.807 (c) 11.672 (d) 24.119

Question 13: If $f(0)=3, f(1)=\frac{\alpha}{2}, f(2)=\alpha$, and the Simpson's rule for $\int_0^2 f(x)\,dx$ gives 2, then the value of α is:

(a) 2.0 (b) 0.5 (c) 1.0 (d) 3.0

Question 14: Given initial-value problem $y' = x+y,\ y(0)=1$, the approximate value of $y(0.1)$ using Euler's method with $n=1$ is:

(a) 1.2 (b) 1.01 (c) 1.02 (d) 1.1

Question 15: Given $y' - \frac{1}{3y} = 0,\ y(0)=1$, the approximate value of $y(1)$ using Taylor's method of order 2 when $n=1$ is:

(a) $\frac{23}{18}$ (b) $\frac{25}{18}$ (c) $\frac{19}{18}$ (d) $\frac{17}{18}$

Question 16: Find the values of a, b and c such that the iterative scheme

$$x_{n+1} = ax_n + \frac{bN}{x_n^2} + \frac{cN^2}{x_n^5}, \qquad n \geq 0,$$

converges at least cubically to $\alpha = N^{\frac{1}{3}}$. Use this scheme to find second approximation of $(27)^{\frac{1}{3}}$ when $x_0 = 2.8$. [5 points]

Question 17: Consider the following nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Find the matrix forms of Jacobi and Gauss-Seidel iterative methods. Show that Gauss-Seidel iterative method converges faster than Jacobi iterative method for the given system. [5 points]