King Saud University: Second Semester Maximum Marks = 40

Mathematics Department 1439-40 H

Math-254 Final Examination Time: 180 mins.

Name of the Student:——————————— I.D. No. ———————

Name of the Teacher:——————————— Section No. ———————

Note: Check the total number of pages are Five (5).
(10 Multiple choice questions and Three (3) Full questions)

The Answer Tables for Q.1 to Q.10 : Marks: 2 for each one ($2 \times 10 = 20$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Quest. No.	Marks
Q. 1 to Q. 10	
Q. 11	
Q. 12	
Q. 13	
Total	

Question 1: The number of bisections required to solve the equation $x^3 + x = 1$ in $[0, 1]$ accurate to within 10^{-3} is:

(a) 8 (b) 10 (c) 9 (d) 7

Question 2: Given $x_0 = 0$ and $x_1 = 1$, then the next approximation x_2 of the solution of the equation $x^4 + 2x = 1$ using the Secant method is:

(a) 0.333 (b) 0.500 (c) 0.250 (d) 0.225

Question 3: The order of multiplicity of the root $\alpha = 0$ of the equation $e^x - \frac{x^2}{2} = x + 1$ is:

(a) 2 (b) 1 (c) 3 (d) 4

Question 4: The goal of forward elimination steps in simple Gauss elimination is to reduce the coefficient matrix to a matrix of the form:

(a) Upper-triangular (b) Lower-triangular (c) Identity (d) Diagonal

Question 5: Let $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $\alpha > 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to:

(a) 3 (b) 5 (c) 4 (d) 6

Question 6: If $f(x) = xe^x$, then $f[0, 1, 0]$ equals to:

(a) $e + 2$ (b) $e + 1$ (c) $e - 2$ (d) $e - 1$

Question 7: Using data points: $(0.0, -2.0), (0.1, -1.0), (0.15, 1.0), (0.2, 2.0), (0.3, 3.0)$, the best approximate value of the function $f(0.11)$ by a linear Lagrange polynomial is:

(a) -0.4 (b) -0.5 (c) -0.6 (d) -0.3

Question 8: If $f(0) = 3, f(1) = \frac{\alpha}{2}, f(2) = \alpha$, and Simpson's rule for $\int_0^2 f(x)\, dx$ gives 2, then the value of α is:

(a) 1.0 (b) 2.0 (c) 0.5 (d) 3.0

Question 9: When using the two-point formula with $h = 0.2$ for approximating the value of $f'(1)$, where $f(x) = \ln(x + 1)$, we have the computed approximation (accurate to 4 decimal places):

(a) 0.4766 (b) 0.4966 (c) 0.4666 (d) 0.4866

Question 10: Given $xy' + y = 1,\ y(1) = 0$, the approximate value of $y(2)$ using Euler's method when $n = 1$ is:

(a) 0.0 (b) 1.0 (c) 2.0 (d) 3.0

Question 11: Consider the following nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Find the matrix form of Jacobi and Gauss-Seidel iterative methods and show that Gauss-Seidel method converges faster than Jacobi method for the given system. [7 points]

Question 12: Given $f(x) = x^{1/3}$, and $x_0 = 0, x_1 = 1, x_2 = 27, x_3 = 64$. Construct the divided differences table for the function. Find the linear splines which interpolate this data. Find the best approximation of $f(8)$ and the absolute error. [7 points]

Question 13: Find the largest value of the step size h (one decimal place) that can be used of to estimate the integral $\int_1^2 \frac{e^{-x}}{x}\, dx$ to an accuracy of 0.5×10^{-2} using the composite Trapezoidal rule. Then, find the corresponding approximate value of the integral. [6 points]