**King Saud University: Mathematics Department MATh-254**
**First Semester 1444 H Final Examination**
**Maximum Marks = 40 Time: 180 mins.**

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**Name of the Student:——————————— I.D. No. ——————————-**

**Name of the Teacher:——————————— Section No. ——————————**

# Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

**The Answer Tables for Q.1 to Q.15 :** Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

| Q. No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a,b,c,d | | | | | | | | | | | | | | | |

| Question No. | Marks Obtained | Marks for Questions |
|---|---|---|
| Q. 1 to Q. 15 | | 30 |
| Q. 16 | | 5 |
| Q. 17 | | 5 |
| Total | | 40 |

**Question 1**: The first approximation of the square root of 19 using a quadratic convergent method with $x_0 = 5$ is:

**(a)** 4.44 **(b)** 4.4 **(c)** 4.36 **(d)** None of these

**Question 2**: The first approximation of the double root of $x^3 - 3x^2 + 4 = 0$, by quadratic convergent iterative method using $x_0 = 1.5$ is:

**(a)** 2.056 **(b)** 2.255 **(c)** 1.250 **(d)** None of these

**Question 3:** The first approximation of the solution of the system of nonlinear equations $4x^3 + y = 6$ and $yx^2 = 1$ using Newton's method with initial approximation $(x_0, y_0) = (1, 1)$ is:

**(a)** $(x_1, y_1) = (0.91, 1.09)$ (b) $(x_1, y_1) = (1.09, 0.91)$ (c) $(x_1, y_1) = (1.5, 0.5)$ (d) None of these

**Question 4**: If the matrix $A = \begin{pmatrix} 3 & 1 \\ 6 & 1 \end{pmatrix}$ is factored as $LU$ using Doollitle's method, where $L$ is a lower triangular matrix, and $U$ is an upper triangular matrix, then the solution of the system $L\mathbf{y} = [-1,\ 0]^T$ is:

**(a)** $[-1,\ -2]^T$ **(b)** $[-1,\ 2]^T$ **(c)** $[-1,\ 6]^T$ **(d)** None of these

**Question 5**: The second approximation for solving linear system $A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ using Jacobi iterative method where $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{x}^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is :

**(a)** $\mathbf{x}^{(2)} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ **(b)** $\mathbf{x}^{(2)} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **(c)** $\mathbf{x}^{(2)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ **(d)** None of these

**Question 6**: The number of iterations needed to achieve accuracy $10^{-4}$ using Gauss-Seidel iterative method if $\|T_G\| = \frac{1}{3}$, $\mathbf{x}^{(0)} = [1, 0, -1]^T$ and $\mathbf{x}^{(1)} = [1.2, 2.3, 3.1]^T$ is :

**(a)** 8 **(b)** 9 **(c)** 11 **(d)** None of these

**Question 7:** Let $x_0 = 2.0,\ x_1 = 3.0,\ x_2 = 4.0$, and $f(x) = x \ln x + e^{-x}$. Then error bound for the approximation of $f(2.5)$ using best Lagrange interpolating polynomial is:

**(a)** 0.25 **(b)** 0.024 **(c)** 0.204 **(d)** None of these

**Question 8:** Absolute error $|f(0.3) - p_1(0.3)|$ in approximating $f(0.3)$ by using linear Lagrange polynomial passing through $x_0 = 0$ and $x_1 = 1$ where $f(x) = x - 10x^2$ is:

**(a)** 0.1 **(b)** 0.05 **(c)** 2.1 **(d)** None of these

**Question 9:** Let $f(x) = e^{-x}$ and $x_0 = 0, x_1 = 0, x_2 = 1$, then the approximation of $1/e^{0.5}$ by using quadratic Newton's polynomial is:

(a) 0.5920 (b) 0.6920 (c) 0.4920 (d) None of these

**Question 10:** Let $f(x) = \ln(2x+1)$ , and $x_0 = 0, x_1 = 1$. Then, the largest possible value of $h$ needed to approximate $f'(x_0)$ accurate to within $10^{-2}$ using these points and the 2-point forward difference formula is approximately:

(a) 0.001 (b) 0.05 (c) 0.005 (d) None of these

**Question 11:** Let $f(x) = x^2 + \cos x$ ($x$ in radian) and $h = 0.1$. Then, using the best 3-point formula for the approximation of $f'(1)$, the absolute error is:

(a) 0.0134 (b) 0.0014 (c) 0.0125 (d) None of these

**Question 12:** Let $f(x)$ be a differentiable function satisfies $f''(x) = (x+1)f(x)$. If $f(0.5) = -1$ and $f(1.5) = 3$, then, using 3-point central difference formula for the second derivative, the approximate value of $f''(1)$ is equals to:

(a) 1.6 (b) 0.8 (c) 0.25 (d) None of these

**Question 13:** Let $f(x) = x\ln(x+1)$. Then, the upper bound for approximating the integral $\int_0^1 f(x)dx$ by using the simple trapezoidal rule is:

(a) 0.0833 (b) 0.0625 (c) 0.1667 (d) None of these

**Question 14:** Let $f(x) = \dfrac{1}{x+1}$. The number of iterations $n$ required composite Simpson's rule to approximate the integral $\int_0^1 f(x)dx$ to within $10^{-3}$ is:

(a) 4 (b) 3 (c) 2 (d) None of these

**Question 15:** If the actual solution of the initial value problem, $y' + y = 2x,\ y(0) = -1, n = 1$, is $y(x) = e^{-x} + 2x - 2$, then the absolute error by using Euler's method of $y(0.1)$ is:

**(a)** 0.0484 **(b)** 0.0289 **(c)** 0.0048 **(d)** None of these

**Question 16:** Use the quadratic Lagrange interpolating polynomial by selecting the best three points from $\{-1, 0.25, 0.5, 1, 2\}$ on the function defined by $f(x) = e^{(x+1)}\cos(x+1)$ ($x$ in radian) to approximate $e^{(1.26)}\cos(1.26)$. Compute the absolute error and an error bound.

**Question 17:** Use the best integration rule to compute the integral $\int_{0.0}^{1.2} f(x)dx$, where the table for the values of $y = f(x)$ is given below:

| $x$ | 0.0 | 0.1 | 0.15 | 0.2 | 0.3 | 0.4 | 0.45 | 0.6 | 0.75 | 0.9 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.0000 | 2.1002 | 2.2015 | 2.3052 | 2.4129 | 2.4688 | 2.6475 | 2.8487 | 3.0812 | 3.2586 | 3.6825 |

The function tabulated is $f(x) = e^x + \cos x$ ($x$ in radian), compute absolute error. How many subintervals approximate the given integral to within accuracy of $10^{-4}$ ?

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| a,b,c,d | a | c | d | a | c | b | a | a | c | b | a | b | b | c | a |

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| a,b,c,d | c | b | d | c | b | a | c | b | b | a | c | c | a | b | b |

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| a,b,c,d | b | a | d | b | a | c | b | c | a | c | b | a | c | a | c |

**Question 16:** Use the quadratic Lagrange interpolating polynomial by selecting the best three points from $\{-1, 0.25, 0.5, 1, 2\}$ on the function defined by $f(x) = e^{(1-x)}\cos(1-x)$ ($x$ in radian) to approximate $e^{(0.74)}\cos(0.74)$. Compute the absolute error and an error bound.

**Solution.** Since the given function is $f(x) = e^{(1-x)}\cos(1-x)$, so by taking $1 - x = 0.74$, we have $x = 0.26$, therefore, the best points for the quadratic polynomial are, $x_0 = 0.25, x_1 = 0.5$, and $x_2 = 1$. Best form of the constructed table for the quadratic Lagrange polynomial is

| $x$ | 0.25 | 0.5 | 1.0 |
|---|---|---|---|
| $f(x)$ | 1.5490 | 1.4469 | 1.0000 |

Then using these table values and the quadratic Lagrange interpolating polynomial

$$f(x) = p_2(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2),$$

$$f(0.26) \approx p_2(0.26) = 1.5490L_0(0.26) + 1.4469L_1(0.26) + 1.0000L_2(0.26).$$

The Lagrange coefficients can be calculate as follows:

$$L_0(0.26) = \frac{(0.26-0.5)(0.26-1)}{(0.25-0.5)(0.25-1)} = 0.9472,$$

$$L_1(0.26) = \frac{(0.26-0.25)(0.26-1)}{(0.5-0.25)(0.5-1)} = 0.0592,$$

$$L_2(0.26) = \frac{(0.26-0.25)(0.26-0.5)}{(1-0.25)(1-0.5)} = -0.0064.$$

Using these values of the Lagrange coefficients, we have

$$f(0.26) \approx p_2(0.26) = 1.5490(0.9472) + 1.4469(0.0592) + 1.0000(-0.0064) = 1.5465,$$

which is the required approximation of the given exact solution $e^{(0.74)}\cos(0.74) = 1.5478$. Thus, we have desired absolute error

$$|f(0.26) - p_2(0.26)| = |1.5478 - 1.5465| = 0.0013.$$

To compute an error bound for the approximation of the given function in the interval $[0.25, 1]$, we use the following quadratic error formula

$$|f(x) - p_2(x)| = \frac{|f^{(3)}(\eta(x))|}{3!}|(x-x_0)(x-x_1)(x-x_2)|.$$

$$|f^{(3)}(\eta(x))| \leq M = \max_{0.25 \leq x \leq 1} |f^{(3)}(x)|,$$

$$\begin{aligned}
f(x) &= e^{(1-x)}\cos(1-x) = e^{(1-x)}\cos(x-1), \qquad (\text{cos(1-x) = cos(x-1)})\\
f'(x) &= -e^{(1-x)}(\cos(x-1) + \sin(x-1)),\\
f''(x) &= 2e^{(1-x)}\sin(x-1),\\
f^{(3)}(x) &= 2e^{(1-x)}(\cos(x-1) - \sin(x-1)).
\end{aligned}$$

Thus

$$M = \max_{0.25 \leq x \leq 1} \left| 2e^{(1-x)}(\cos(x-1) - \sin(x-1)) \right| = 5.9840,$$

at $x = 0.25$. Hence

$$|f(0.26) - p_2(0.26)| \leq \frac{5.9840}{6}|(0.26-0.25)(0.26-0.5)(0.26-1)| \leq \frac{5.9840}{6}(0.0018) = 0.0017952,$$

which is the desired error bound. •

**Question 17:** Use the best integration rule to compute the integral $\int_{0.0}^{1.2} f(x)dx$, where the table for the values of $y = f(x)$ is given below:

| $x$ | 0.0 | 0.1 | 0.15 | 0.2 | 0.3 | 0.4 | 0.45 | 0.6 | 0.75 | 0.9 | 1.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2.0000 | 2.1002 | 2.2015 | 2.3052 | 2.4129 | 2.4688 | 2.6475 | 2.8487 | 3.0812 | 3.2586 | 3.6825 |

The function tabulated is $f(x) = e^x + \cos x$, compute absolute error. How many subintervals approximate the given integral to within accuracy of $10^{-4}$ ?

**Solution.** Since the equally spaced data points are for $h = 0.3$,

| $x$ | 0.0 | 0.3 | 0.6 | 0.9 | 1.2 |
|---|---|---|---|---|---|
| $f(x)$ | 2.0000 | 2.4129 | 2.8487 | 3.2586 | 3.6825 |

which gives, $n = 4$, so the best integration rule is the composite Simpson's rule and which is

$$\int_{x_0}^{x_4} f(x)dx \approx S_4(f) = \frac{h}{3}\Big[f(x_0) + 4(f(x_1) + f(x_3)) + 2f(x_2) + f(x_4)\Big].$$

Using the table values, we get

$$\int_{0.0}^{1.2} (e^x + \cos x)dx \approx S_4(f) = 0.1\Big[2.0000 + 4(2.4129 + 3.2586) + 2(2.8487) + 3.6825\Big] = 3.4066.$$

$$Exact\ Solution = \int_{0.0}^{1.2} (e^x + \cos x)dx = (e^x + \sin x)\Big|_0^{1.2} = 3.2522.$$

Thus the absolute error $|E|$ in our approximation is given as

$$|E| = |3.2522 - S_4(f)| = |3.2522 - 3.4066| = 0.1544.$$

The fourth derivative of the function $f(x) = e^x + \cos x$ can be obtain as

$$f'(x) = e^x - \sin x, \quad f''(x) = e^x - \cos x, \quad f'''(x) = e^x + \sin x, \quad f^{(4)}(x) = e^x + \cos x.$$

The bound $|f^{(4)}(x)|$ on $[0, 1.2]$ is

$$M = \max_{0 \le x \le 1.2} |f^{(4)(x)}| = \max_{0 \le x \le 1.2} |e^x + \cos x| = 3.6825,$$

at $x = 1.2$. To find the minimum subintervals for the given accuracy, we use

$$|E_{S_n}(f)| \le \frac{(b-a)^5}{180n^4} M \le 10^{-4},$$

where $h = (1.2 - 0)/n$. Since $M = 3.6825$, then solving for $n^4$,

$$n^4 \ge \frac{(b-a)^5 M 10^4}{180} = \frac{(1.2-0)^5(3.6825)10^4}{180} = 509.0688,\ n^2 \ge 22.5626,\ n \ge 4.75,\ n = 6(even).$$

or

$$h^4 \le \frac{180}{(b-a)M10^4} = \frac{180}{(1.2-0)^5(3.6825)10^4} = 0.0041,\ h^2 \le 0.0638,\ h \le 0.2526,\ n = (1.2-0)/0.2526 = 4.75,$$

so n = 6(even).