King Saud University
College of Science
Department of Mathematics

244 Math
Second Midterm Exam
Two Hours

Name:	**Student No.:**
Section / Group No.:	**Sequence No.:**

Question No.	**I**	**II**	**III**	**IV**	**V**	**Total**
Mark						

I. Determine whether the following statements are always true or sometimes false, and justify your answer with a logical argument or a counter example:

(a) If A and B are two square matrices of the same size with $\det(AB)=0$ then $\det(A)=0$.

☐ True ☐ False

Justification:

(b) $T\,(x\,,y\,)=(0,0)$ defines a linear operator on $\mathbb{R}^2$ but $T\,(x\,,y\,)=(1,1)$ does not.

☐ True ☐ False

Justification:

(c) If A is a square matrix whose entries are all integers and $\det(A)=1$ then all the entries of A^{-1} are integers.

☐ True ☐ False

Justification:

(d) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are two vectors in $\mathbb{R}^n$ such that $\|\boldsymbol{u}-\boldsymbol{v}\|=0$ then $\boldsymbol{u}=\boldsymbol{v}$.

☐ True ☐ False

Justification:

(e) If A is a 3×3 matrix with $\det(A)=k$, k scalar, then $\det(\text{Adj}A)=\frac{1}{k}$.

☐ True ☐ False

Justification:

(f) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are two vectors in $\mathbb{R}^n$ such that $\boldsymbol{u}\cdot\boldsymbol{v}=0$ then $\|\boldsymbol{u}+\boldsymbol{v}\|+\|\boldsymbol{u}-\boldsymbol{v}\|=0$.

☐ True ☐ False

Justification:

II. Choose the correct answer:

(a) If $A=\begin{bmatrix} k-1 & -2 \\ -6 & k-2 \end{bmatrix}$, then A is not invertible if:

i. $k=1$ or $k=2$.

ii. $k=-1$ or $k=-4$.

iii. $k=-2$ or $k=5$.

iv. $k=3$ or $k=8$.

(b) If $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(x+2y,-x+y)$ is a one to one linear operator then $\left[T^{-1}\right]$ is equal to:

i. $\begin{bmatrix} -1 & 2 \\ -1 & -1 \end{bmatrix}$.

ii. $\begin{bmatrix} 1/3 & -2/3 \\ 1/3 & 1/3 \end{bmatrix}$.

iii. $\begin{bmatrix} 1/3 & 2/3 \\ -1/3 & 1/3 \end{bmatrix}$.

iv. $\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$.

(c) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are two orthogonal vectors in $\mathbb{R}^n$ such that $\|\boldsymbol{u}\|=\|\boldsymbol{v}\|=3$ then $d(\boldsymbol{u},\boldsymbol{v})$ is equal to:

i. 9.

ii. 18.

iii. 0.

iv. $3\sqrt{2}$.

(d) In $\mathbb{R}^2$, the standard matrix of a reflection about the x-axis followed by a rotation through an angle θ is:

i. $\begin{bmatrix} -\cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

ii. $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

iii. $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$.

iv. $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$.

(e) For any vector $\boldsymbol{u}$ in $\mathbb{R}^n$ and any scalar k, if $\|k\boldsymbol{u}\| = -k\,\|\boldsymbol{u}\|$ then:

i. $k \geq 0$.

ii. $k \leq 0$.

iii. $k \neq 0$.

iv. None of the above.

III. Use row operations and cofactor expansion to evaluate $\det(A)$ where:

$$A = \begin{bmatrix} 0 & 2 & 5 & 2 \\ -1 & 0 & -2 & 0 \\ 3 & -1 & -3 & 2 \\ 6 & 0 & 10 & 0 \end{bmatrix}$$

IV. Use Cramer's rule to solve for x_2 without solving for x_1, x_3, and x_4.

$$\begin{array}{rcrcrcrcr} & & 2x_2 & + & 5x_3 & + & 2x_4 & = & -15 \\ x_1 & & & + & 2x_3 & & & = & -6 \\ 3x_1 & - & x_2 & - & 3x_3 & + & 2x_4 & = & 9 \\ 6x_1 & & & + & 10x_3 & & & = & -30 \end{array}$$

V. Determine whether the linear operator $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by the equations is one to one; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2, w_3)$.

$$\begin{array}{rcrcrcr} w_1 & = & x_1 & - & 2x_2 & + & x_3 \\ w_2 & = & 2x_1 & + & x_2 & + & x_3 \\ w_3 & = & x_1 & & & + & x_3 \end{array}$$

Good Luck