First Semester	**Second Exam**	**King Saud University**
(without calculators)	**Time allowed: 1 h and 30 m**	**College of Science**
Tuesday 4-3-1437	**240 Math**	**Math. Department**

Q1: (a) Let V be any vector space which has two operations are defined: addition and scalar multiplication. State the 10 axioms that should be satisfied by all scalars and all objects in V. (5 marks)

(b) Prove that the solution set of a homogeneous linear system Ax=0 in n unknowns is a subspace of $\mathbb{R}^n$. (2 marks)

Q2: (a) Use the wronskian to show that 1, x, x^2 are linearly independent. (2 marks)

(b) show that the vectors (1,2,1), (2,2,2), (3,4,0) form a basis for $\mathbb{R}^3$. (3 marks)

Q3: (a) Let B={(1,3),(0,1)} and B'={(1,1),(2,1)} be two basis of $\mathbb{R}^2$. Find the transition matrix from B' to B. (3 marks).

(b) Find a basis for the column space of the matrix:

$$A=\begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 5 & 9 & -4 & 8 \end{bmatrix}$$

and **<u>deduce</u>** dim(null(A)) without solving any linear system. (3 marks)

Q4: (a) Show that the matrix operator T from $\mathbb{R}^2$ to itself defined by the equations:

$w_1 = 2x_1 + x_2$

$w_2 = 3x_1 + 4x_2$

is 1-1 and find T^{-1}. (3 marks)

(b) Show that $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ is diagonalizable and find a matrix P that diagonalizes A. (4 marks)