FINAL EXAMINATION, SEMESTER II: 1438-1439
DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 240 FULL MARK: 40 TIME: 3 HOURS

1. (a) Decide if the vectors $\mathbf{v}_1 = (1,0,1,2)$, $\mathbf{v}_2 = (2,1,1,1)$ and $\mathbf{v}_3 = (1,1,0,-1)$ in $\Re^4$ form a linearly dependent or independent set. **Marks**: 4
(b) Find the coordinate vector for $\mathbf{v} = (5,-12,3)$ relative to the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\Re^3$, where $\mathbf{v}_1 = (1,2,3)$, $\mathbf{v}_2 = (-4,5,6)$, $\mathbf{v}_3 = (7,-8,9)$. **Marks**: 3

2. Find eigenvalues and eigenvectors of the the matrix A:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

Is the matrix diagonalizable? Explain in detail. **Marks**: 2+2+3=7

3. Check whether the vectors $\mathbf{w}_1 = (0,2,0)$, $\mathbf{w}_2 = (3,0,3)$, $\mathbf{w}_3 = (-4,0,4)$ form an orthogonal basis for $\Re^3$ with the Euclidean inner product; if so, then use that basis to find an orthogonal basis by normalizing each vector. **Marks**: 4+3=7

4. Assume that the vector space $\Re^4$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where $\mathbf{v}_1 = (1,1,1,1)$, $\mathbf{v}_2 = (3,1,-1,1)$, $\mathbf{v}_3 = (1,1,3,3)$ into an orthogonal basis $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$, and then normalize the orthogonal basis vectors to obtain an orthogonal basis $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$. **Marks**: 5+2=7

5. Find all least squares solutions of the linear system. **Marks: 4**

$$\begin{aligned} 2x_1 - 2x_2 &= 1 \\ x_1 + x_2 &= -1 \\ 2x_1 + x_2 &= 1 \end{aligned}$$

6. (a) Let

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 6 & -4 \\ 7 & 4 & 2 \end{bmatrix}$$

Find the rank and nullity of T. **Marks**: 2+2=4
(b) Let $T\colon \Re^3 \longrightarrow \Re^3$ be multiplication by A, where

$$A = \begin{bmatrix} 2 & 6 & 10 \\ 0 & 1 & -1 \\ 2 & 4 & 6 \end{bmatrix}$$

Determine whether T has an inverse; if so, find $T^{-1}\left([x_1\ x_2\ x_3]^t\right)$. **Marks**: 3+1=4