FIRST SEMESTER FINAL EXAMINATION, 1439-1440(DEC. 2018)
DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 240 FULL MARK: 40 TIME: 3 HOURS

[**N. B.: All questions carry equal mark** $5 \times 8 = 40$]

1. (a) Determine whether $\mathbf{v_1} = (1,2,6)$, $\mathbf{v_2} = (3,4,1)$, $\mathbf{v_3} = (4,3,1)$, and $\mathbf{v_4} = (3,3,1)$ span the vector space $\Re^3$.
(b) Check whether the set of vectors $\mathbf{v_1} = (3,8,7,-3)$, $\mathbf{v_2} = (1,5,3,-1)$, $\mathbf{v_3} = (2,-1,2,6)$, and $\mathbf{v_4} = (1,4,0,3)$ in $\Re^4$ is linearly dependent or independent.

2. Find a subset of the vectors $\mathbf{v_1} = (1,-1,5,2)$, $\mathbf{v_2} = (-2,3,1,0)$, $\mathbf{v_3} = (4,-5,9,4)$, $\mathbf{v_4} = (0,4,2,-3)$ and $\mathbf{v_5} = (-7,18,2,-8)$ that forms a basis for the space spanned by these vectors.

3. Find a basis for the orthogonal complement of the subspace of $\Re^n$ spanned by the vectors $\mathbf{v_1} = (1,4,5,2)$, $\mathbf{v_2} = (2,1,3,0)$, $\mathbf{v_3} = (-1,3,2,2)$.

4. Assume that the vector space $\Re^3$ has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$, where $\mathbf{u_1} = (1,0,0)$, $\mathbf{u_2} = (3,7,-2)$, $\mathbf{u_3} = (0,4,1)$ into an orthogonal basis $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.

5. Find the characteristic equation of the following matrix and hence find eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

6. Find a matrix P that diagonalizes A, and determine $P^{-1}AP$.

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

7. Let T be multiplication by the matrix A, where

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$

Find the rank and nullity of T.

8. Let $T\colon \Re^3 \longrightarrow \Re^3$ be defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_1, x_1 - x_3)$.
Find the matrix for T with respect to the basis $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where $\mathbf{v_1} = (1,0,1)$, $\mathbf{v_2} = (0,1,1)$ and $\mathbf{v_3} = (1,1,0)$. Hence verify that $[T]_B[\mathbf{x}]_B = [T(\mathbf{x})]_B$, for every vector $\mathbf{x} = (x_1, x_2, x_3)$ in $\Re^3$.