1st Semester 1437/1438	Final Exam	King Saud University
(without calculators)	Time allowed: 3 hours	College of Science
Wednesday 13-4-1438	240 Math	Math. Department

Q1: Suppose the reduced row echelon form (R. R. E. F.) of a matrix A is $R=\begin{bmatrix}1&0&0&0\\0&1&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$:

(a) Find The solution set of the system $Ax=0$.

(b) If the columns of A are v_1, v_2, v_3, v_4. Find a basis of the column space of A from the set $\{v_1, v_2, v_3, v_4\}$. (4 marks)

Q2: Let V be a subspace of the vector space $\mathbb{R}^3$ **spanned** by the set S, where S={ v_1=(3,-1,0), v_2=(2, -3,4), v_3=(-1, 5, 1), v_4=(1,2,3), v_5=(7,0,7)}. Find a **subset** of S that forms a **basis** of V. (4 marks)

Q3: Find a basis for each eigenspace of the matrix $A=\begin{bmatrix}1&1&0\\0&1&0\\0&1&1\end{bmatrix}$. Moreover, find the algebraic multiplicity and the geometric multiplicity of each eigenvalue of A and **deduce** if the matrix A is diagonalizable or not. (5 marks)

Q4: Let $\mathbb{R}^4$ be the Euclidean inner product space. Find the distance between the vectors u=(2,2,3,0) and v=(1,1,2,-1). Also, show that these two vectors are **not** orthogonal. (4 marks)

Q5: Assume that the vector space $\mathbb{R}^3$ has the Euclidean inner product. Apply the **Gram-Schmidt process** to transform the basis vectors (1,0,1), (0,1,2), (0,3,0) into an **orthonormal** basis. (6 marks)

Q6: Let $c \in \mathbb{R}-\{0\}$ and V be an inner product space, and let $T:V\to V$ be the map defined by $T(v)=cv$ for all v in V. Show that:

(a) T is a linear operator.

(b) If $v_o \in \ker(T)$, then $v_o=0$. (5 marks)

Q7: Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by the formula: $T(x_1, x_2, x_3) = (3x_1, -2x_1 - 4x_2, 3x_1 + 4x_2 - 2x_3)$. Find $[T]_{S,B}$ where S is the standard basis of $\mathbb{R}^3$ and B={v_1=(1,1,1),v_2=(1,1,0),v_3=(1,0,0)} is another basis of $\mathbb{R}^3$. (4 marks)

Q8: Solve the following statements:

(a) If $T:V \to W$ is a linear transformation, then prove that the range of T (R(T)) is a subspace of W. (2 marks)

(b) If $T_1:U \to V$ and $T_2:V \to W$ are two linear transformations, then prove that $(T_2 \circ T_1):U \to W$ is also a linear transformation. (2 marks)

(c) If u and v are orthogonal vectors in an inner product space, then prove that: $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ (2 marks)

(d) Define a product on the vector space M_{22} as follows:

for all $A, B \in M_{22} : \langle A, B \rangle = |AB|$

Show that this product is **<u>not</u>** an inner product on M_{22}. (2 marks)