

KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
TIME: 1H 30 mn, FULL MARKS: 40, 22/01/1432
MATH 204

Question 1. a) [7] Find the largest interval for which the following initial value problem has a unique solution

$$\begin{cases} x^2 y'' + \frac{x}{\sqrt{2-x}} y' + \frac{2}{\sqrt{x}} y = 0 \\ y(1) = 2, \quad y'(1) = 1 \end{cases}$$

b) [7] If the function $y_1 = e^{-x}$ is a solution of the differential equation

$$(x^2 + 2)y'' - 2xy' - (x^2 + 2x + 2)y = 0.$$

Use formula to find the second solution y_2 , and hence find the general solution.

Question 2. a) [9] Find the general solution of the nonhomogeneous differential equation:

$$y'' + y' - 2y = \cos x.$$

b) [6]. Determine only the form of the particular solution of the differential equation

$$y'' + 2y' + 10y = x^2 e^{-x} \sin 3x.$$

Question 3. [9] By using the transformation $x = e^t$, find the general solution of the Cauchy-Euler equation

$$x^2 y'' + xy' + 4y = \cos(\ln x^2), \quad x > 0.$$

Question 4. [9] Solve the system of differential equations

$$\begin{cases} \frac{dx}{dt} - 4x - y = 2 \\ \frac{dy}{dt} + 2x - y = 0. \end{cases}$$

NOTE: For question 1, answer either part a) or part b).