King Saud University,
College of Sciences
Mathematical Department.

Mid-Term Exam/S3/2023
Full Mark:30. Time 2H
03/05/2023

Question 1. [5,4] **a)** Determine and sketch the largest local region of the $xy-$plane for which the initial value problem

$$\begin{cases} \ln(9-x^2)\frac{dy}{dx} = \sqrt{y-3} + \cos y \\ y(0) = 5, \end{cases}$$

has a unique solution.

b) Find the general solution of the differential equation

$$(x+y)dy + \sqrt{x^2+y^2+2xy}dx = 0, \quad x > 0, \ y > 0$$

Question 2. [4, 4]. **a)** Solve the differential equation

$$\left(\frac{y^2}{x} + 2x\ln y\right)dx + \left(\frac{x^2}{y} + 2y\ln x\right)dy = 0, \ x > 0, \ y > 0.$$

b) Obtain the general solution of the differential equation

$$(x^3 + 3y^3)dx - 3xy^2dy = 0, \quad x > 0.$$

Question 3. [4, 4]. **a)** Solve the initial value problem

$$\begin{cases} (y\sin x + \sin x)dx = (2-\cos x)(y^2+y+1)dy, \ y \neq -1 \\ y(0) = 0. \end{cases}$$

b) By using an appropriate substitution, solve the initial value problem

$$\begin{cases} (y-2x)y' = (1+y-2x)^2 \\ y(0) = 4\sqrt{3}. \end{cases}$$

Question 4. [5] The number of bacteria in a culture is doubled in 4 hours. If the number of bacteria after 8 hours in 2000. What will be the initial number of bacteria. The rate of growth of bacteria is proportional to the number of bacteria at present time.