

Question 1 [4,4] a) Discuss the existence of unique solution of the following initial value problem

$$\begin{cases} (x-2)y'' + \frac{x}{\sqrt{3-x}}y' + \frac{1}{x^2-4}y = \cos x \\ y(1) = 0, y'(1) = 1. \end{cases}$$

b) Solve the nonhomogeneous differential equation

$$y'' - 2y' - 3y = e^{2x} + 5 \cos 2x$$

Question 2 [4,3]. a) If $y_1 = x^3 e^x$ is a solution of the differential equation

$$xy'' - 2(x+1)y' + (x+2)y = 0, \quad x \neq 0,$$

then use reduction of order method to obtain its general solution.

b) Determine either the functions

$$f_1(x) = e^{2x}, \quad f_2(x) = e^{-2x}, \quad f_3(x) = \cosh 2x,$$

are linearly independent or linearly dependent on $(-\infty, \infty)$.

Question 3 [5] Find the general solution of the differential equation

$$x^2 y'' - 3xy' + 3y = x^4 e^x; \quad x > 0.$$

Question 4 [5] Solve the following linear system of differential equations.

$$\begin{cases} x' = x - y + t \\ y' = x + 3y - 3t \end{cases}$$