Question 1 [4,4] a) Find the general solution of the differential equation

$$[\sin(x+y) + y\cos(x+y) + x + y]\, y' + (y\cos(x+y) + y + x) = 0$$

b) Solve the initial value problem

$$\begin{cases} \dfrac{x^2 y'}{y^4} + \dfrac{x}{y^3} = \sin x, & x > 0. \\ y(5) = 3. \end{cases}$$

Question 2 [4,4] a) Find the general solution of the differential equation

$$\left(\ln(2y - 8)\cos x + \frac{1}{x} \right) dx = -\frac{\sin x}{y - 4}\, dy, \qquad 0 < x < \pi,\ y > 4.$$

b) Obtain the general solution of the following differential equation

$$\left(\frac{(x+y)^3}{3x} + (x+y)^2 \right) dx + (x+y)^2\, dy = 0, \quad x > 0,\ x + y \neq 0.$$

Question 5 [6]. Find the family of orthogonal trajectories for the family of curves

$$y = e^{C\sin x}.$$

Question [4,4]. a) Find and sketch the largest local region of the xy−plane for which the initial value problem

$$\begin{cases} y\, dx + x(\ln x - \ln y - 1)dy = 0, & x > 0,\ y > 0. \\ y(1) = e, \end{cases}$$

has a unique solution

b) Find the solution of the differential equation:

$$\frac{dy}{dx}(y - 1)\sqrt{x^2 + 1} + x^3 + x = y(x^3 + x), \qquad y \neq 1.$$