Question 1[4,4]. a) Find and sketch the largest region of the $xy-$plane for which the initial value problem

$$\begin{cases} \sqrt{x^2 - 1}\, dx - \frac{1}{\ln(y-1)}dy = 0 \\ y(3) = 3/2. \end{cases}$$

has a unique solution.

   b) Solve the initial value problem

$$\begin{cases} y' - xe^{-x}\sin(x) + y = 0 \\ y(0) = 1 \end{cases}$$

Question 2[4,4]. a) Solve the following differential equation

$$\frac{dy}{dx} = \frac{yx\sin(x) - x^2 y}{-xy^2 + yxe^{-3y}}, \quad y > 0, \; x > 0.$$

b) Find the general solution of the differential equation

$$[(x\ln(2y - 8)\cos x + 1)](4 - y)dx = x\sin x\, dy, \quad 0 < x < \pi, \; y > 4.$$

Question 3[4]. Solve the differential equation

$$(y - xy)dy - (x + y^2)dx = 0, \quad x > 1.$$

Question 4[5]. A certain culture of bacteria grows at a rate proportional to its size. let P_0 be the initial size. If the size doubles in 4 days and the size becomes $3P_0 + 750$ in 12 days. Find P_0.