King Saud University College of Since Mathematical Department Differential equations MATH204	جامعة الملك سعود King Saud University 1957	Mid-Term 1 Summer Semester 1440 Time 1H30min Full Mark:25

Question 1: [4] Find and sketch the largest region of the xy-plane for which the initial value problem

$$\begin{cases} (1+y^3)\dfrac{dy}{dx} = x^2 \\ y(0) = 0, \end{cases}$$

has a unique solution.

Question 2: [4, 4]

a) Solve the initial value problem

$$\begin{cases} (xy^2 + 4x)dx + (8y - 2x^2y)dy = 0, & |x| < 2 \\ y(0) = 0 \end{cases}$$

b) Obtain the general solution of the differential equation

$$2cosx\frac{dy}{dx} + ysinx - (4x + 5)^2y^3 = 0, \qquad x \in (\frac{-\pi}{2}, \frac{\pi}{2})$$

Question 3:[4, 4]

a) Solve the differential equation

$$(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$$

b) By using an appropriate substitution, or any other method, find the genera solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x}\ln(xy), \qquad x > 0, \qquad y > 0$$

Question 4: [5] A small metal bar, whose initial temperature was 20°C, is dropped into a large container of boiling water. One second later the object's temperature is 22°C. How long will it take the bar to reach 90°C? How long will it take the bar to reach 98°C?