

Question 1[4,4]. a) Find the region about $(1, -2)$ of the xy -plane for which the initial value problem

$$\begin{cases} y' = \sqrt{1-2y} \ln x \\ y(1) = -2. \end{cases}$$

has a unique solution.

b) Solve the initial value problem

$$\begin{cases} xdx = (1+x^2)^2 \sin y dy \\ y(-1) = \pi/2. \end{cases}$$

Question 2[4,4]. a) Show that the following differential equation is exact, hence solve it

$$(3y \cos x - xe^x)dx + (3 \sin x + 3)dy = 0.$$

b) Find the general solution of the differential equation

$$(x + \sqrt{xy})dy - ydx = 0 \quad x > 0, \quad y > 0.$$

Question 3[4,4]. a) Solve the differential equation

$$y' + y \cot x = x \csc x, \quad 0 < x < \pi.$$

b) Determine the general solution of

$$y' + y = xy^3.$$

Question 4[5]. Find the family of orthogonal trajectories for the family of curves

$$2y + x = Ce^{2y}.$$

Remark: Answer a) or b) in question 3.