M – 204 DEPARTMENT OF MATHEMATICS FULL MARKS:50
KING SAUD UNIVERSITY TIME: 90 min
(FIRST MID-TERM SEMESTER I, summer 1436)

Question:1. Find the general solutions to the differential equations

(i) $\dfrac{dy}{dx} = \dfrac{x+y+1}{x+y-1}$

(ii) $\dfrac{dy}{dx} = \dfrac{y^2+1}{y(x^2+1)}$ **[10]**

Question:2. For the differential equation

$$xydx + (a\,x^2 + 3y^2 - 20)dy = 0, \quad a \in R\,. \qquad \textbf{[10]}$$

(a) Find ***a*** such that the differential equation is exact.
(b) Hence solve the obtained differential equation.

Question:3. Find the largest region of the plane for which the initial value problem

$\sqrt{x^2-9}\dfrac{dy}{dx} = x \ln y, \quad y(4) = 5$ has a unique solution. **[10]**

Question:4. Write the differential equation in the form of Bernoulli's equation. hence solve it $2xdy + (8y^3 - xy - y)dx = 0, \quad x > 0$. **[10]**

Question: 5. The population of a town grows at a rate proportional to the population present at time t. The initial population of 50000 increases by 10% in five years. What will be population after 20 years? [Formulate the differential equation and then solve]. **[10]**