King Saud University,
College of Sciences
Mathematical Department.

Mid-Term1 /S2/2018
Full Mark:25. Time 1H30mn
08/03/2018

Question 1[5]. Find and sketch the largest region of the xy−plane for which the initial value problem

$$\begin{cases} (x^2 - x - 2) \cdot \frac{dy}{dx} = \sqrt{1 - \ln(2 - y)} \\ y(0) = 0, \end{cases}$$

has a unique solution.

Question 2[4+4]. **a)** Solve the initial value problem

$$\begin{cases} (xy^2 + 4x)dx + (8y - 2x^2y)dy = 0, \quad |x| < 2. \\ y(0) = 0. \end{cases}$$

b) Obtain the general solution of the differential equation

$$2\cos x \frac{dy}{dx} + y\sin x - (4x + 5)^2 y^3 = 0, \quad x \in (\frac{-\pi}{2}, \frac{\pi}{2}).$$

Question 3[4+4]. **a)** Solve the differential equation

$$(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0.$$

b) By using an appropriate substitution, or any other method, find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x}\ln(xy), \quad x > 0, \quad y > 0.$$

Question 4[4]. Let $y(1 + \cos x) = C$, where $x \in (0, \pi)$, be a given family of curves. Find the the family of orthogonal trajectories for this family of curves.

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