King Saud University,
College of Sciences
Mathematical Department.

Mid-Term1 /S1/2016
Full Mark:25. Time 1H30mn
02/02/1438

Question 1[4,4]. **a)** Determine the local region in the xy−plane for which the following differential equation

$$\sqrt{9-y^2}\frac{dy}{dx} = \ln(4-x^2),$$

would have a unique solution through the origin $(0,0)$.

b) Find the solution of the differential equation:

$$(x^2-x-2)\frac{dy}{dx} = (x-2)^2 + 3y, \quad x > 2.$$

Question 2[4,4]. **a)** Verify that the differential equation

$$(x^2+y^2-2)dx + (x^2-2xy)dy = 0, \quad x(x-2y) \neq 0.$$

is not exact. Find a suitable integrating factor to convert it to an exact equation, and then solve it.

b) Solve the initial value problem

$$\begin{cases} 5xy^2y' + y^3 = 32(1+\ln x)y^{-2}, & x > 0, \ y \neq 0 \\ y(1) = 1 \end{cases}$$

Question 3[4]. Solve the differential equation

$$\frac{dy}{dx} = \frac{1-x-y}{x+y}, \quad x+y \neq 0.$$

Question 4[5]. Initially 100 mg of a radioactive substence was present. After 8 hours the mass has decreased by 4%. If the rate of decay is proportional to the amount of the substence present at time t. Find the amount of the remaining after 50 hours.