

**Question 1[4,4].** a) Find and sketch the largest local region of the  $xy$ -plane for which the initial value problem

$$\begin{cases} ydx + x(\ln x - \ln y - 1)dy = 0, & x > 0, y > 0. \\ y(1) = e, \end{cases}$$

has a unique solution.

b) Show that  $\mu(x, y) = xy$  is an integrating factor for the following differential equation, and solve it

$$(-xy \sin x + 2y \cos x)dx + 2x \cos x dy = 0.$$

**Question 2[4+4].** a) Solve the initial value problem

$$\begin{cases} y \frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{1-x^2}, & y \neq 0, x \neq \pm 1 \\ y(0) = 2. \end{cases}$$

b) Find the general solution of the differential equation

$$(xy^2 - y)dx + xdy = 0, x > 0.$$

**Question 3[4+4].** a) Solve the differential equation

$$(1 + x - y - xy)dx + (1 - y^2)e^y dy = 0, x \neq -1, y \neq 1.$$

b) Find the solution of the initial value problem

$$\begin{cases} (x \sin^2 x + \frac{\cos x}{\sin x} y)dx - dy = 0 \\ y(\frac{\pi}{2}) = 0, 0 < x < \pi \end{cases}$$

**Question 4[6].** Find the family of orthogonal trajectories for the family of curves:  $y = C \sin x$ .