$y2\cos x\,(4y+\ln y)$

King Saud University, Department of Mathematics
Math 204 (3H), 40/40, Final Exam 04/01/2018

Question 1[4,4] a) Solve the differential equation

$$(8y^2\cos x + 2y\cos x.\ln y)dx - (4y+1)(3+2\sin x)dy = 0, \quad y > 0.$$

b) Solve the initial value problem

$$2(y-1)dx + (x^2-1)dy = 0, \quad y(0) = 0, \quad -1 < x < 1.$$

Question 2[4,4] a) Determine the value of K so that the following differential equation is exact and solve it.

$$(y^3 + Kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0, \quad x \neq 0, y \neq 0.$$

b) A radioactive substance has a half life of 1620 years. If the rate of decay is proportional to the amount of substance present at any time, then how many grams of a sample of 120 grams of this substance will be left after 100 years.

Question 3[4,5,5] a) Find only the form of the particular solution y_p of the differential equation

$$y^{(3)} - 4y' = 2x + 4x\cos x + 3xe^{-2x}.$$

b) Find the first three non-zero terms of the power series solution for the differential equation

$$(1+x^2)y'' - xy' + y = 0,$$

about the ordinary point $x_0 = 0$.

c) Solve the initial value problem

$$\begin{cases} y'' - 2y' + y = \frac{e^x}{x}, & x > 0, \\ y(1) = e, \quad y'(1) = 0 \end{cases}$$

Question 4[5,5] a) Compute the Fourier cosine series for the function: $f(x) = \sin x,\ x \in (0, \pi)$.

Deduce the value of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1}$.

b) Sketch the graph of the following function and find its Fourier integral

$$f(x) = \begin{cases} \pi & \text{if} \quad |x| \leq 1 \\ 0, & \text{if} \quad |x| > 1 \end{cases}.$$

Deduce the value of the integral $\int_0^{\infty} \frac{\sin 2\lambda}{\lambda} d\lambda$.