Department of Mathematics

King Saud University

Second Mid Term Exam. Second Semester (1434/35)

M-203

N.B: <u>All questions carry equal marks</u>

Time: 90 Minutes | Max. Marks: 25

Question No: 1 Evaluate the integral $\int_0^1 \int_{4y}^4 e^{-x^2} dxdy$.

Question No: 2 Use polar coordinates to evaluate

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin(x^2+y^2) dydx .$$

Question No: 3 Find the area of the surface $z=\sqrt{a^2-x^2-y^2}$ that lies in the first octant.

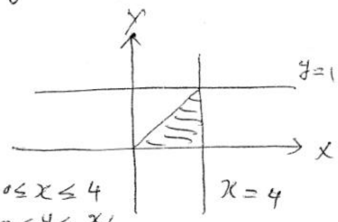
Question No: 4 Find the volume of the solid bounded by

$$y=x^2, y+z=4, \text{ and } z=0.$$

Question No: 5 Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} z \, dzdydx$ by using Spherical coordinates.

Q:1 [5]

$$\int_0^1 \int_{4y}^{4} e^{-x^2}\,dx\,dy \qquad 4 \qquad 0 \le y \le 1,\quad 4y \le x \le 4$$



$$= \int_0^4 \int_0^{x/4} e^{-x^2}\,dy\,dx = \int_0^4 e^{-x^2}\, \tfrac{x}{4}\,dx \qquad 4$$

$0 \le x \le 4$

$0 \le y \le x/4$

$$= -\frac{1}{8}\int_0^4 e^{-x^2}(-2x)\,dx = -\frac{1}{8}\left[e^{-x^2}\right]_0^4$$

$$= -\frac{1}{8}\left[e^{-16} - 1\right] \qquad 1$$

Q:2 [5]

$$\int_{-3}^{3}\int_0^{\sqrt{9-x^2}} \sin(x^2+y^2)\,dy\,dx = \int_0^{\pi}\int_0^3 \sin(r^2)\,r\,dr\,d\theta \qquad 2+1$$

$$= \frac{1}{2}\int_0^{\pi}\int_0^3 \sin(r^2)\,2r\,dr\,d\theta = \frac{1}{2}[\cos 9 - 1]\int_0^{\pi} d\theta \;\; 2 \;\; = \frac{1}{2}[\cos 9 - 1]\,\pi$$

Q:3 [5]

$$f_x = \frac{-x}{\sqrt{a^2-x^2-y^2}}, \quad f_y = \frac{-y}{\sqrt{a^2-x^2-y^2}} \qquad 2$$

$$S.A. = \iint_{R_{xy}} \sqrt{\frac{a^2}{a^2-x^2-y^2}}\,dA = a\int_0^{\pi/2}\int_0^a \frac{1}{\sqrt{a^2-r^2}}\,r\,dr\,d\theta \qquad 2$$

$$= -\frac{a}{2}\int_0^{\pi/2}\int_0^{a} (a^2-r^2)^{-1/2}(2r)\,dr\,d\theta = -\frac{a}{2}\int_0^{\pi/2}\left[\frac{(a^2-r^2)^{1/2}}{1/2}\right]_0^a d\theta$$

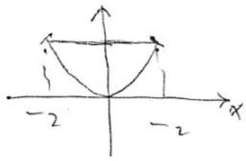
$$= -a\left[0 - (a^2)^{1/2}\right]\left(\frac{\pi}{2}\right) = \frac{a^2\pi}{2} \qquad 1$$

Q. 4
[5]

$$V = \int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-y} dz\,dy\,dx \quad (3)$$

$-2 \le x \le 2$
$x^2 \le y \le 4$
$0 \le z \le 4-y$

$$= \int_{-2}^{2}\int_{x^2}^{4} (4-y)\,dy\,dx$$



$$= \int_{-2}^{2}\left[4y - \frac{y^2}{2}\right]_{x^2}^{4} dx = \int_{-2}^{2}\left[(16-8) - \left(4x^2 - \frac{x^4}{2}\right)\right] dx$$

$$= \left[8x - \frac{4}{3}x^3 + \frac{x^5}{10}\right]_{-2}^{2} = \left[16 - \frac{32}{3} + \frac{32}{10}\right] - \left[-16 + \frac{32}{3} - \frac{32}{10}\right]$$

$$= 32 - \frac{64}{3} + \frac{64}{10}$$

$$= \frac{512}{30} = \frac{256}{15} \quad (2)$$

Q. 5
[5]

$$= \int_{0}^{\pi/2}\int_{0}^{\pi/4}\int_{0}^{1} \rho\cos\varphi\,\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta \quad (3)$$

$0 \le \rho \le 1$
$0 \le \varphi \le \pi/4$
$0 \le \theta \le \pi/2$

$$= \int_{0}^{\pi/2}\int_{0}^{\pi/4} \left.\frac{\rho^4}{4}\right]_{0}^{1} \sin\varphi\cos\varphi\,d\varphi\,d\theta$$

$$= \frac{1}{4} \times \cancel{\frac{1}{2}} \int_{0}^{\pi/2} \left.\frac{\sin^2\varphi}{2}\right]_{0}^{\pi/4} d\theta = \frac{1}{8}\left(\frac{1}{2}\right)\int_{0}^{\pi/2} d\theta = \frac{1}{32}\pi$$

(2)