

Department of Mathematics  
King Saud University  
Second Mid Term Exam. Second Semester (1434/35)  
M-203

N.B: All questions carry equal marks

Time: 90 Minutes

Max. Marks: 25

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Question No: 1 Evaluate the integral  $\int_0^1 \int_{4y}^4 e^{-x^2} dx dy$ .

Question No: 2 Use polar coordinates to evaluate

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2) dy dx.$$

Question No: 3 Find the area of the surface  $z = \sqrt{a^2 - x^2 - y^2}$  that lies in the first octant.

Question No: 4 Find the volume of the solid bounded by

$$y = x^2, y + z = 4, \text{ and } z = 0.$$

Question No: 5 Evaluate the integral  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} z dz dy dx$  by using Spherical coordinates.

$$Q_{1.1} \int_0^1 \int_{4y}^4 e^{-x^2} dx dy \quad (1)$$

[5]

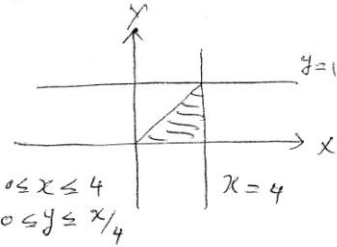
$$= \int_0^4 \int_0^{x/4} e^{-x^2} dy dx = \int_0^4 e^{-x^2} x/4 dx$$

$$= -\frac{1}{8} \int_0^4 e^{-x^2} (-2x) dx = -\frac{1}{8} [e^{-x^2}]_0^4$$

$$= -\frac{1}{8} [e^{-16} - 1] \quad (1)$$

$$0 \leq y \leq 1$$

$$4y \leq x \leq 4$$



$$Q_{1.2} \int_{-3}^3 \int_0^{\sqrt{9-x^2}} \sin(x^2+y^2) dy dx = \int_0^\pi \int_0^3 \sin(r^2) r dr d\theta \quad (2+1)$$

[5]

$$= \frac{1}{2} \int_0^\pi \int_0^3 \sin(r^2) 2r dr d\theta = \frac{1}{2} [\cos \theta - 1] \int_0^\pi d\theta = \frac{1}{2} [\cos \theta - 1] \pi$$

$$Q_{1.3} f_x = \frac{-x}{\sqrt{a^2-x^2-y^2}}, \quad f_y = \frac{-y}{\sqrt{a^2-x^2-y^2}} \quad (2)$$

[5]

$$S.A. = \iint_{R_{xy}} \sqrt{\frac{a^2}{a^2-x^2-y^2}} dA = a \int_0^{\pi/2} \int_0^a \frac{1}{\sqrt{a^2-r^2}} r dr d\theta$$

$$= -\frac{a}{2} \int_0^{\pi/2} \left[ (a^2-r^2)^{-1/2} (2r) \right]_0^a d\theta = -\frac{a}{2} \left[ \frac{(a^2-r^2)^{1/2}}{1/2} \right]_0^a d\theta$$

$$= -a [0 - (a^2)^{1/2}] \left( \frac{\pi}{2} \right) = \frac{a^2 \pi}{2} \quad (1)$$

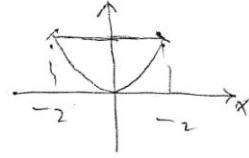
(2)

Q. 4  
[5]

$$V = \int_{-2}^2 \int_{x^2}^4 \int_0^{4-y} dz dy dx \quad (3)$$

$$\begin{aligned} -2 \leq x \leq 2 \\ x^2 \leq y \leq 4 \\ 0 \leq z \leq 4-y \end{aligned}$$

$$= \int_{-2}^2 \int_{x^2}^4 (4-y) dy dx$$



$$= \int_{-2}^2 \left[ 4y - \frac{y^2}{2} \right]_{x^2}^4 dx = \int_{-2}^2 \left[ (16-8) - (4x^2 - \frac{x^4}{2}) \right] dx$$

$$= \left[ 8x - \frac{4}{3}x^3 + \frac{x^5}{10} \right]_{-2}^2 = \left[ 16 - \frac{32}{3} + \frac{32}{10} \right] - \left[ -16 + \frac{32}{3} - \frac{32}{10} \right]$$

$$= 32 - \frac{64}{3} + \frac{64}{10}$$

$$= \frac{512}{30} = \frac{256}{15} \quad (2)$$

Q. 5  
[5]

$$= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^1 \rho \cos \varphi \rho^2 \sin \varphi d\rho d\varphi d\theta \quad (3)$$

$$\begin{aligned} 0 \leq \rho \leq 1 \\ 0 \leq \varphi \leq \pi/4 \\ 0 \leq \theta \leq \pi/2 \end{aligned}$$

$$= \int_0^{\pi/2} \int_0^{\pi/4} \left[ \frac{\rho^4}{4} \right]_0^1 \sin \varphi \cos \varphi d\varphi d\theta$$

$$= \frac{1}{4} \times \int_0^{\pi/2} \left[ \frac{\sin 2\varphi}{2} \right]_0^{\pi/4} d\theta = \frac{1}{8} \left( \frac{1}{2} \right) \int_0^{\pi/2} d\theta = \frac{1}{32} \pi$$

(2)