

King Saud University  
Department of Mathematics  
M-203  
(Differential and Integral Calculus)  
Second-Mid Term Examination  
(First Semester 1435/1436)

Max. Marks: 25

Time: 90 minutes

Marking Scheme: All questions carry equal marks.

Q. No:1 Reverse the order of integration and evaluate the resulting integral  $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} dy dx$ .

Q. No:2 Use polar coordinates to evaluate the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx.$$

Q. No:3 Find the surface area of the part of the solid cut off from the sphere  $z = \sqrt{25 - x^2 - y^2}$  by the planes  $z = 3$  and  $z = 4$ .

Q. No:4 Use cylindrical coordinates to evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \int_0^{\sqrt{x^2+y^2}} e^z dz dy dx$$

Q. No:5 Evaluate the integral (by using spherical coordinates)

$$\iiint_Q \sqrt{x^2 + y^2 + z^2} dv,$$

where Q is the solid region bounded by the graphs of the equations

$$z = \sqrt{x^2 + y^2} \quad \text{and} \quad z = \sqrt{2 - x^2 - y^2}.$$

Q: 1

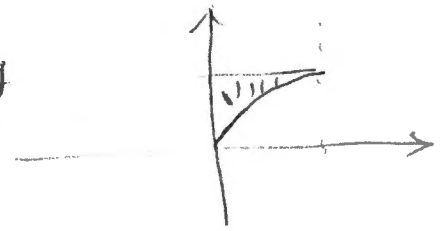
$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1} \, dy \, dx$$

[5 Marks]

$$0 \leq x \leq 1$$

$$\sqrt{x} \leq y \leq 1$$

$$= \int_0^1 \int_0^{y^2} \sqrt{y^3+1} \, dx \, dy = \int_0^1 (y^3+1)^{1/2} y^2 \, dy$$



$$= \frac{1}{3} \left[ \frac{(y^3+1)^{3/2}}{3/2} \right]_0^1 = \frac{2}{9} \left[ (2)^{3/2} - 1 \right]$$

(1)

$$0 \leq y \leq 1$$

$$0 \leq x \leq y^2$$

Q: 2

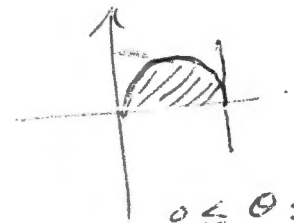
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2} \, dy \, dx$$

[5 Marks]

$$0 \leq x \leq 2$$

$$0 \leq y \leq \sqrt{2x-x^2}$$

$$= \int_0^{\pi/2} \int_0^{2\cos\theta} r (r \, dr \, d\theta)$$



$$0 \leq \theta \leq \pi/2$$

$$0 \leq r \leq 2\cos\theta$$

$$= \int_0^{\pi/2} \left[ \frac{r^3}{3} \right]_0^{2\cos\theta} d\theta = \frac{8}{3} \int_0^{\pi/2} \cos^3\theta \, d\theta$$

$$= \frac{8}{3} \int_0^{\pi/2} (1 - \sin^2\theta) \cos\theta \, d\theta$$

$$u = \sin\theta$$

$$du = \cos\theta \, d\theta$$

$$= \frac{8}{3} \int_0^1 (1 - u^2) \, du = \frac{8}{3} \left[ u - \frac{u^3}{3} \right]_0^1$$

$$= \frac{8}{3} \left[ 1 - \frac{1}{3} \right] = \frac{16}{9}$$

(1)

(2)

Q.3

$$S.A. = \iint_{R_{xy}} \sqrt{1 + f_x^2 + f_y^2} \, dA \quad (1)$$

$$= \iint_{R_{xy}} \frac{5}{\sqrt{25-x^2-y^2}} \, dA$$

$$= \int_0^{2\pi} \int_3^4 \frac{5}{\sqrt{25-r^2}} \, r \, dr \, d\theta \quad (2)$$

$$= 5 \int_0^{2\pi} \int_3^4 (25-r^2)^{-1/2} (r) \, dr \, d\theta = -\frac{5}{2} \int_0^{2\pi} \left. \frac{(25-r^2)^{1/2}}{1/2} \right|_3^4 \, d\theta$$

$$= -5 [3-4] \int_0^{2\pi} d\theta = 10\pi \quad (1)$$

[5 Marks]

$$f_x = \frac{-x}{\sqrt{25-x^2-y^2}}$$

$$f_y = \frac{-y}{\sqrt{25-x^2-y^2}} \quad (1)$$

$$z=3, z=4$$

$$\boxed{3 \leq r \leq 4}$$

$$\boxed{0 \leq \theta \leq 2\pi}$$

Q.4

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \int_0^{\sqrt{x^2+y^2}} e^z \, dz \, dy \, dx$$

$$= \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} \int_0^r e^z \, r \, dz \, dr \, d\theta \quad (3)$$

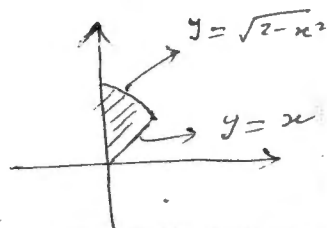
$$= \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} \left. e^z \right|_0^r \, r \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} r(e^r - 1) \, dr \, d\theta = \int_{\pi/4}^{\pi/2} \left[ \int_0^{\sqrt{2}} r e^r \, dr - \int_0^{\sqrt{2}} r \, dr \right] d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left[ \left. \{r e^r - e^r\} \right|_0^{\sqrt{2}} - \left. \left\{ \frac{r^2}{2} \right\} \right|_0^{\sqrt{2}} \right] d\theta = \int_{\pi/4}^{\pi/2} \left[ \sqrt{2} e^{\sqrt{2}} - e^{\sqrt{2}} + 1 - 1 \right] d\theta$$

$$= (\sqrt{2} e^{\sqrt{2}} - e^{\sqrt{2}}) \left[ \frac{\pi}{4} \right] = e^{\sqrt{2}} (\sqrt{2} - 1) \frac{\pi}{4} \quad (2)$$

[5 Marks]



$$\boxed{x \leq y \leq \sqrt{2-x^2}}$$

$$\boxed{0 \leq x \leq 1}$$

$$\boxed{0 \leq y \leq \sqrt{2}}$$

$$\boxed{\pi/4 \leq \theta \leq \pi/2}$$

$$\boxed{0 \leq r \leq y}$$

(3)

Q.5

$$\iiint_Q \sqrt{x^2 + y^2 + z^2} \, dV$$

[5 MARKS]

$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{2}} \rho \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta \quad \text{3+1=4}$$

$$= \int_0^{2\pi} \int_0^{\pi/4} \left[ \frac{\rho^4}{4} \right]_0^{\sqrt{2}} \sin \varphi \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} \left[ -\cos \varphi \right]_0^{\pi/4} d\theta = \int_0^{2\pi} \left[ -\frac{1}{\sqrt{2}} + 1 \right] d\theta$$

$$= \left[ -\frac{1}{\sqrt{2}} + 1 \right] 2\pi = \sqrt{2}\pi(\sqrt{2} - 1) \quad \text{①}$$

