

King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(I-Semester 1434/1435)

Max. Marks: 25

Time: 90 Minutes

Marking Scheme: Q.1:(4), Q.2:(4), Q.3:(4), Q.4:(4), Q.5:(4), Q.6:(5)

Q. No: 1 Reverse the order of integration, and evaluate the resulting integral

$$\int_0^4 \int_{\sqrt{y}}^2 y \cos(x^5) dx dy.$$

Q. No: 2 Use polar coordinates to evaluate the integral

$$\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2)^{3/2} dy dx, \text{ where } a > 0.$$

Q. No: 3 Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ lying between the planes $x = 0$, $y = 0$, $z = 0$, and $z = 4$.

Q. No: 4 Consider the lamina over the region R in the first quadrant bounded by the curves $y = x^3$ and $y = x$ with area mass density $\delta(x, y) = y$. Find the moment of inertia of the lamina about the x -axis.

Q. No: 5 Find the volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 16$ and the xy -plane.

Q. No: 6 Use spherical coordinates to evaluate the integral

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$$

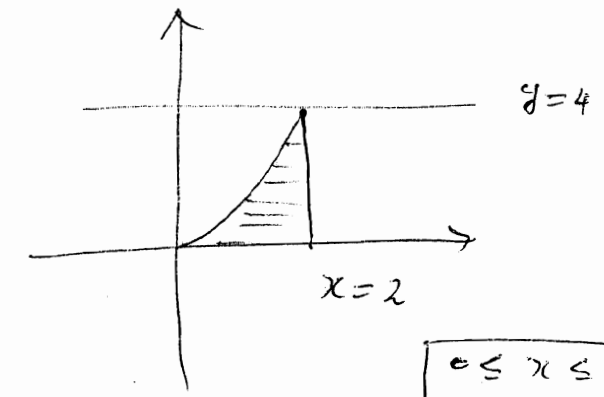
Q:1

[Marks: 4]
$$= \int_0^2 \int_0^{x^2} y \cos(x^5) dy dx$$

(2)
$$= \frac{1}{2} \int_0^2 x^4 \cos(x^5) dx$$

$$= \frac{1}{10} \int_0^{32} \cos u du = \frac{1}{10} \sin u \Big|_0^{32} + c$$

$$= \frac{1}{10} \sin(32) + c$$



$$\begin{aligned} 0 &\leq x \leq 2 \\ 0 &\leq y \leq x^2 \end{aligned}$$

Put $u = x^5$
 $\frac{1}{5} du = x^4 dx$

(2)

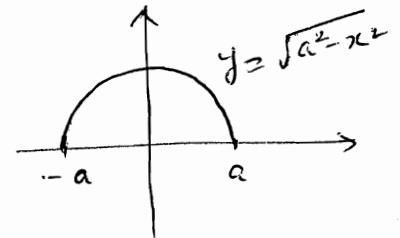
Q:2

[Marks: 4]
$$\int_{-a}^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2} dy dx$$

(2)
$$= \int_0^\pi \int_0^a r^3 r dr d\theta = \int_0^\pi \left[\frac{r^5}{5} \right]_0^a d\theta$$

$$= \frac{a^5}{5} \int_0^\pi d\theta = \frac{a^5 \pi}{5}$$

(1)



$$\begin{aligned} 0 &\leq r \leq a \\ 0 &\leq \theta \leq \pi \end{aligned}$$

Q:3

S.A =
$$\iint \sqrt{1+f_x^2+f_y^2} dA$$

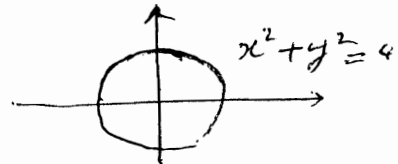
$z = f(x,y) = x^2 + y^2$

[Marks: 6]
$$\frac{1}{\sqrt{2}} \int_0^{2\pi} \int_0^2 \sqrt{1+(2x)^2+(2y)^2} r dr d\theta$$

(3)
$$= \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2} r dr d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} \left[\frac{(1+4r^2)^{3/2}}{3/2} \right]_0^2 d\theta = \frac{1}{12} [17^{3/2} - 1] (2\pi)$$

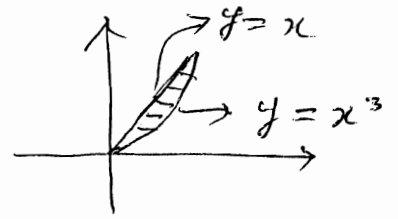
(1)



Q. 4

[Marks: 4]

$$I_x = \iint_R \delta y^2 dA = \iint_R y^3 dA$$



$$I_x = \int_0^1 \int_{x^3}^x y^3 dy dx = \frac{1}{4} \int_0^1 [y^4]_{x^3}^x dx$$

(2)

$$0 \leq x \leq 1 \\ x^3 \leq y \leq x$$

$$= \frac{1}{4} \int_0^1 [x^4 - x^{12}] dx = \frac{1}{4} \left[\frac{x^5}{5} - \frac{x^{13}}{13} \right]_0^1$$

$$= \frac{1}{4} \left[\frac{1}{5} - \frac{1}{13} \right] = \frac{1}{4} \frac{8}{65} = \frac{2}{65}$$

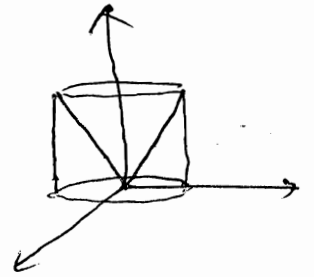
(1)

Q. 5

$$V = \iiint dV$$

Cylindrical

$$0 \leq z \leq r \\ 0 \leq r \leq 4 \\ 0 \leq \theta \leq 2\pi$$



[Marks: 4]

$$= \int_0^{2\pi} \int_0^4 \int_0^r r dz dr d\theta$$

(3)

$$= \int_0^{2\pi} \int_0^4 \frac{r^2}{2} dr d\theta = \int_0^{2\pi} \left[\frac{r^3}{3} \right]_0^4 d\theta = \frac{64}{3} (2\pi) = \frac{128\pi}{3}$$

Spherical

$$0 \leq \rho \leq 4 \csc \varphi$$

$$\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{2}$$

$$0 \leq \theta \leq 2\pi$$

$$V = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^{4 \csc \varphi} \rho^2 \sin \varphi d\rho d\varphi d\theta = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \left[\frac{\rho^3}{3} \right]_0^{4 \csc \varphi} \sin \varphi d\varphi d\theta$$

(3)

$$= \frac{1}{3} \int_0^{2\pi} \int_{\pi/4}^{\pi/2} 64 \csc^3 \varphi \sin \varphi d\varphi d\theta = \frac{64}{3} \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \csc^2 \varphi d\varphi d\theta$$

$$= \frac{64}{3} \int_0^{2\pi} [-\cot \varphi]_{\pi/4}^{\pi/2} d\theta = \frac{64}{3} \int_0^{2\pi} [-0 + 1] d\theta = \frac{128}{3} \pi$$

(1)

#

(3)

Q.6
 [Marks: 5]
$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2) dz dy dx$$

④
$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^2 \rho^2 \rho^2 \sin \varphi d\rho d\varphi d\theta$$

$0 \leq \rho \leq 2$
 $0 \leq \varphi \leq \pi/4$
 $0 \leq \theta \leq 2\pi$

$$= \int_0^{2\pi} \int_0^{\pi/4} \left[\frac{\rho^5}{5} \right]_0^2 \sin \varphi d\varphi d\theta = \frac{32}{5} \int_0^{2\pi} [-\cos \varphi]_0^{\pi/4} d\theta$$

$$= \frac{32}{5} \left[-\frac{1}{\sqrt{2}} + 1 \right] \int_0^{2\pi} d\theta$$

$$= \frac{64}{5} \left(1 - \frac{1}{\sqrt{2}} \right) \pi$$

① #