King Saud University
Department Of Mathematics
M-203
(Differential and Integral Calculus)
First Mid-Term Examination
(II-Semester 1436/1437)

Max. Marks: 25 Time: 90 Minutes

Q. No: 1 Determine whether the series $\sum_{n=1}^{\infty}\left[2^{-n}+2^{-3n}\right]$ is convergent or divergent. If it is convergent, find its sum. [4]

Q. No: 2 Test the convergence or divergence of the series

$$\sum_{n=1}^{\infty}\left[\frac{1}{n^{1/3}+n^{1/5}}\right]$$ [5]

Q. No: 3 Find the interval of convergence and radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{2^n(x-2)^n}{n}$. [6]

Q. No:4 Find the power series representation of the function $f(x)=\frac{x}{8-x^3}$ in powers of x and use the first two non-zero terms of this power series to approximate the integral $\int_0^1 \frac{x}{8-x^3}dx$. [5]

Q, No: 5 Find the Maclaurin series of $\cos x$ and use it to find the Maclaurin series of $f(x)=\cos(x^2)$. [5]

Q.1 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=1}^{\infty} \frac{1}{2^{3n}}$ [[illegible] Marks]

(2)

$$= \frac{1/2}{1-1/2} + \frac{1/8}{1-1/8} = \frac{1/2}{1/2} + \frac{1/8}{7/8}$$

(2)

$$= 1 + \frac{1}{7} = \frac{8}{7}$$

Q:2 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/3} + n^{1/5}}$ Comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

[5 Marks]

$$\frac{a_n}{b_n} = \frac{1/n^{1/3}}{\frac{1}{n^{1/3} + n^{1/5}}} = \frac{n^{1/3} + n^{1/5}}{n^{1/3}} = \frac{1 + \frac{n^{1/5}}{n^{1/3}}}{1}$$

$$= 1 + n^{-2/15}$$

$\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1 \neq 0$ both series c'gc or d'gc together

(3)

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/3}+n^{1/5}}$ is d'gt

(2)

Q:3 $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{n}$

[6 Marks]

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{2^{n+1}(x-2)^{n+1}}{n+1} \times \frac{n}{2^n (x-2)^n}\right| = \frac{2n}{n+1}|x-2|$$

$\displaystyle\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = 2|x-2| \Rightarrow$ c'gt if $2|x-2| < 1$

$$\Rightarrow |x-2| \leq 1/2 \Rightarrow -1/2 < x-2 < 1/2$$

$$\Rightarrow 3/2 < x < 5/2$$

At $x = 3/2$

$$\sum_{n=1}^{\infty} \frac{2^n (3/2 - 2)^n}{n}$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

By AST c'gt (1)

At $x = 5/2$

$$\sum_{n=1}^{\infty} \frac{2^n (5/2 - 2)^n}{n}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{d'st}$$ (1)

Interval of c'gence $\quad 3/2 \le x < 5/2$

Radius of convergence $r = \frac{1}{2}\left[\frac{5}{2} - \frac{3}{2}\right] = \frac{1}{2}\left[\frac{5-3}{2}\right] = \frac{1}{2}$ (1)

Q:4 [5 Marks]

$$f(x) = \frac{x}{8 - x^3} = \frac{x}{8}\left[\frac{1}{1 - x^3/8}\right] \quad \text{if } \left|\frac{x^3}{8}\right| < 1$$

$$= \frac{x}{8}\left[1 + \left(\frac{x^3}{8}\right) + \left(\frac{x^3}{8}\right)^2 + \cdots + \left(\frac{x^3}{8}\right)^n + \cdots\right]$$ (3)

$$\int_0^1 \frac{x}{8}\left[1 + \frac{x^3}{8}\right] dx \approx \int_0^1 \left[\frac{x}{8} + \frac{x^4}{8}\right] dx = \left.\frac{x^2}{16} + \frac{x^5}{40}\right]_0^1$$

$$= \frac{1}{16} + \frac{1}{40} = \frac{5 + 2}{80} = \frac{7}{80} = 0.0875$$ (2)

Q:5 [5 Marks]

$$f(x) = \cos x \Rightarrow f(0) = \cos(0) = 1$$

$$f'(x) = -\sin x \Rightarrow f'(0) = -\sin(0) = 0$$

$$f''(x) = -\cos x \Rightarrow f''(0) = -1$$

$$f'''(x) = \sin x \Rightarrow f'''(0) = 0 \Rightarrow f^{iv}(0) = 1$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$ (3)

Replacing x by x^2

$$\cos(x^2) = 1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \cdots + (-1)^n \frac{(x^2)^{2n}}{(2n)!} + \cdots$$ (2)