King Saud University
Department Of Mathematics
M-203
(Differential and Integral Calculus)
First Mid-Term Examination
(II-Semester 1435/1436)

Max. Marks: 25 Time: 90 Minutes

Q. No: 1 Determine whether or not the sequence $\left\{\dfrac{n^2+n+1}{5^n}\right\}_{n=1}^{\infty}$ converges, and if it converges find its limit. [4]

Q. No: 2 Test the convergence or divergence of the series

$$\sum_{n=1}^{\infty}\left[\frac{1}{n^{1/n}}\right]$$ [4]

Q. No: 3 Determine whether the series $\displaystyle\sum_{n=2}^{\infty}\frac{cos(n\pi)}{n^2\ln(n)}$ is convergent or divergent. [5]

Q. No: 4 Find the interval of convergence and radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{4^n n^2}$. [6]

Q, No: 5 Find the Maclaurin series for $f(x)=e^x$ and use the first two non-zero terms of the Maclaurin series of $\sqrt{e^x}$ to approximate the integral

$$\int_0^1 \frac{\sqrt{e^x}-1}{x}dx.$$ [6]

Q: 1

$$\underset{n\to\infty}{Lt} \frac{n^2+n+1}{5^n} \quad \frac{\infty}{\infty} \quad 1 \qquad [Marks: 4]$$

$$= \underset{x\to\infty}{Lt} \frac{2x+1}{5^x \ln 5} \quad \frac{\infty}{\infty} \quad 1$$

$$= \underset{x\to\infty}{Lt} \frac{2}{5^x(\ln 5)^2} = 0 \; ; \text{ convergent.}$$

1 1

Q: 2 $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$. Applying the nth term test [Marks: 4]

$$\underset{n\to\infty}{Lt} \frac{1}{n^{1/n}} = \frac{1}{\underset{n\to\infty}{Lt} n^{1/n}} = \frac{1}{1} = 1 \neq 0 . \text{ Hence d'gt.}$$

3 1

Q: 3 $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{n^2(\ln n)}$. First check A.C. [Marks: 5]

$$\sum_{n=2}^{\infty} \left| \frac{\cos n\pi}{n^2 \ln n} \right| = \sum_{n=2}^{\infty} \frac{|\cos n\pi|}{n^2 \ln n} \quad . \text{ Note } |\cos n\pi| \leq 1$$

Now using B.C.T $\sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} b_n$. c'gt p-series

3

$$\frac{|\cos n\pi|}{n^2 \ln n} \leq \frac{1}{n^2} \text{ for } n \geq 2 \Rightarrow \text{c'gt}$$

2

Hence given series is ~~A~~ Convergent.

②

Q:4 $\sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}$. [Marks: 6]

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{4^{n+1}(n+1)^2} \times \frac{4^n n^2}{x^n} \right| = \lim_{n\to\infty} \frac{n^2}{4(n+1)^2} |x| = \frac{1}{4}|x|$$

③

$\Rightarrow$ c'gt if $|x| < 4 \Rightarrow \boxed{-4 < x < 4}$

check at $x = -4$ ①

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{4^n n^2} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

By A.S.T it is c'gt

check at $x = 4$ ①

$$\sum_{n=1}^{\infty} \frac{4^n}{4^n n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

It is a c'gt p-series

$\Rightarrow$ Interval of c'gence $-4 \le x \le 4$: $[-4, 4]$

and radius of c'gence $= \rho = \frac{4-(-4)}{2} = 4$ ①

Q:5 $f(x) = e^x,\ f'(x) = e^x, \ldots, f^{(n)}(x) = e^x, \ldots$ [Marks: 6]

$f(0) = 1,\ f'(0) = 1, \ldots, f^{(n)}(0) = 1, \ldots$ ②

Maclaurin Series is

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sqrt{e^x} = e^{x/2} = 1 + (x/2) + \frac{1}{2!}(x/2)^2 + \cdots$$ ②

$$e^{x/2} - 1 = \frac{x}{2} + \frac{x^2}{8} + \cdots$$

$$\int_0^1 \frac{e^{x/2}-1}{x}\,dx \approx \int_0^1 [\tfrac{1}{2} + x/8 + \cdots]\,dx \approx \left.\frac{x}{2} + \frac{x^2}{16}\right]_0^1 = \frac{1}{2} + \frac{1}{16}$$

$$= \frac{9}{16} = 0.5625$$ ②