King Saud University
Department of Mathematics
M-203
(Differential & Integral Calculus)
First Mid-Term Examination
(II-Semester 1434/35)

Max. Marks: 25 **Time: 90 minutes**

Marks: Q.1[4]; Q.2[5]; Q.3[4]; Q.4[6]; Q.5[6]

Q.No: 1 Determine whether or not the sequence $\left\{\left(\frac{n+3}{n+1}\right)^n\right\}_{n=1}^{\infty}$ converges, and if it converges find its limit.

Q. No: 2 Find the **sum** of the series $\sum_{n=1}^{\infty}\left[\frac{2}{n(n+1)}+\left(\frac{e}{3}\right)^n\right]$.

Q. N0: 3 Determine whether the following series is **absolutely convergent**, **conditionally convergent**, or **divergent** $\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln n)^n}$.

Q.No: 4 Find the interval of convergence and the radius of convergence of the power series: $\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{3^n(n+1)}$.

Q. N0: 5 Find the power series representation for the function f(x) = $\ln(1+x)$ and

i) approximate $\ln(1.03)$ by using first two non-zero terms of this series;

ii) estimate the error in this approximation.

Q:1 [Mark : 4]

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(n+3)^n}{(n+1)^n} = \lim_{n\to\infty} \frac{(1+3/n)^n}{(1+1/n)^n} \quad 2$$

$$= \frac{\lim_{n\to\infty} (1+3/n)^n}{\lim_{n\to\infty} (1+\frac{1}{n})^n} = \frac{e^3}{e} = e^2 \quad 2 \quad \#$$

Q. 2 [Mark : 5]

$$\sum_{n=1}^{\infty} \left[\frac{2}{n(n+1)} + \left(\frac{e}{3}\right)^n\right] = \sum_{n=1}^{\infty} \frac{2}{n(n+1)} + \sum_{n=1}^{\infty} \left(\frac{e}{3}\right)^n$$

$$= \sum_{n=1}^{\infty} \left[\frac{2}{n} - \frac{2}{n+1}\right] + \frac{e/3}{1-e/3}$$

$$\boxed{\begin{aligned} s_n &= \left[\frac{2}{1} - \cancel{\frac{2}{2}}\right] + \left[\cancel{\frac{2}{2}} - \cancel{\frac{2}{3}}\right] + \dots + \left[\cancel{\frac{2}{n}} - \frac{2}{n+1}\right] \\ \lim_{n\to\infty} s_n &= 2 \end{aligned}} \quad 2$$

$$2$$

$$= 2 + \frac{e/3}{(3-e)/3} = 2 + \frac{e}{3-e} \quad 1 \quad \#$$

Q:3 [Mark: 4]

$$\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{(\ln n)^n}\right| = \sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$$

Applying the root test 2

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1 \Rightarrow \text{C'gt.}$$

Given Series is A.C. 2

Q:4 $\displaystyle \underset{n\to\infty}{Lt} \left|\frac{u_{n+1}}{u_n}\right| = \underset{n\to\infty}{Lt} \left|\frac{x^{n+1}}{3^{n+1}(n+2)} \times \frac{3^n(n+1)}{x^n}\right|$

[Mark: 6]

$$= \underset{n\to\infty}{Lt} \frac{(n+1)}{3(n+2)} |x| = \frac{1}{3}|x|$$

C'gt if $\frac{1}{3}|x| < 1 \Rightarrow |x| < 3 \Rightarrow -3 < x < 3$ (3)

C'gence at $x=-3$

$$\sum_{n=0}^{\infty} \frac{(-1)^n(-3)^n}{3^n(n+1)}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n+1} \quad \text{d'g}$$ (1)

C'gence at $x=3$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{3^n(n+1)}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1}$$

By A.S.T it is c'gt (1)

Interval of C'gence $-3 < x \le 3$

radius " " $= \rho = \frac{3-(-3)}{2} = 3$ (1)

Q:5 $\displaystyle \ln(1+x) = \int_0^x \frac{1}{1+t}\,dt = \int_0^x [1-t+t^2-t^3+\cdots]\,dt$ (1)

[Mark: 6]

$$= \left[t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots\right]_0^x$$ (2)

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots$$

$$\ln(1+0.03) \approx (0.03) - \frac{(0.03)^2}{2} = 0.03 - \frac{9}{10000} \approx 0.0291$$ (2)

$$\text{error} \le \frac{(0.03)^3}{3}$$ (1)