

King Saud University
Department Of Mathematics
M-203
(Differential and Integral Calculus)

First Mid-Term Examination
(I-Semester 1435/36)

Max. Marks: 25

Time: 90 Minutes

Q. No: 1 Determine whether or not the sequence

$$\left\{ n \left(\sqrt{n^2 + 2} - \sqrt{n^2 - 3} \right) \right\}_{n=1}^{\infty}$$

converges, and if it converges find its limit.....[4]

Q. No: 2 Determine whether the following infinite series converges or diverges.
If it converges, find its sum

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} \dots\dots\dots[5]$$

Q. No: 3 Test whether the following series is absolutely convergent, conditionally convergent, or divergent

$$\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n^2} \right) \dots\dots\dots[4]$$

Q. No: 4 Find the interval of convergence and radius of convergence of the

$$\text{power series } \sum_{n=1}^{\infty} \frac{4^n}{\sqrt{n}} x^n \dots\dots\dots [6]$$

Q. No: 5 Find Maclaurin series for $f(x) = \sin x$ and use the first three non-zero terms to approximate

$$\int_0^1 x \sin(x^2) dx \dots\dots\dots[6]$$

①

Q:1 $a_n = n \frac{(\sqrt{n^2+2} - \sqrt{n^2-3}) \times (\sqrt{n^2+2} + \sqrt{n^2-3})}{(\sqrt{n^2+2} + \sqrt{n^2-3})}$ (2)

$$= n \frac{(n^2+2) - (n^2-3)}{\sqrt{n^2+2} + \sqrt{n^2-3}} = n \frac{5}{\sqrt{n^2+2} + \sqrt{n^2-3}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{5}{\sqrt{1 + \frac{2}{n^2}} + \sqrt{1 - \frac{3}{n^2}}} = \frac{5}{2}$$
 (2)

Q:2 $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} = \sum_{n=2}^{\infty} \left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)} \right]$ (2)

$$\begin{aligned} S_n &= \left[\frac{1}{2(1)} - \frac{1}{2(3)} \right] + \left[\frac{1}{2(2)} - \frac{1}{2(4)} \right] + \left[\frac{1}{2(3)} - \frac{1}{2(5)} \right] \\ &\quad + \left[\frac{1}{2(4)} - \frac{1}{2(6)} \right] + \dots + \left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)} \right] \\ &= \frac{1}{2} + \frac{1}{4} + \left[\frac{1}{2(n-1)} - \frac{1}{2(n+1)} \right] \rightarrow \frac{3}{4} \end{aligned}$$
 (1)

Q:3 First a.c. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2} \right)$

use nth term test

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2} \right) = 1$$

\Rightarrow d'gt

Now using AST (ii) $\lim_{n \rightarrow \infty} a_n = 1 \neq 0$ (3)

So given series is d'gt. (1)

(2)

Q:4
$$\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{4^{n+1} x^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{4^n x^n} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{4\sqrt{n}}{\sqrt{n+1}} |x| = 4|x|$$

(2)

$$\text{c'gt} \Rightarrow 4|x| < 1 \Rightarrow |x| < \frac{1}{4} \Rightarrow -\frac{1}{4} < x < \frac{1}{4}$$

check c'gence at $x = -\frac{1}{4}$

$$\sum_{n=1}^{\infty} \frac{4^n (-\frac{1}{4})^n}{\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

By AST it is c'st

(1)

check c'gence at $x = \frac{1}{4}$

$$\sum_{n=1}^{\infty} \frac{4^n (\frac{1}{4})^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

d'gt p-series

(1)

Interval of c'gence $-\frac{1}{4} \leq x < \frac{1}{4}$ (1)

Radius of c'gence $\rho = \frac{\frac{1}{4} + \frac{1}{4}}{2} = \frac{1}{4}$ (1)

Q:5

$$f(x) = \sin x \Rightarrow f(0) = 0$$

$$f'(x) = \cos x \Rightarrow f'(0) = 1$$

$$f''(x) = -\sin x \Rightarrow f''(0) = 0$$

$$f'''(x) = -\cos x \Rightarrow f'''(0) = -1$$

⋮

Maclaurin Series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sin x$ (3)

$$\int_0^1 x \left[x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots \right] dx \approx \int_0^1 \left[x^3 - \frac{x^7}{3!} + \frac{x^{10}}{5!} - \dots \right] dx$$

$$= \left[\frac{x^4}{4} - \frac{x^8}{48} + \frac{x^{11}}{1320} - \dots \right]_0^1 = \frac{1}{4} - \frac{1}{48} + \frac{1}{1320} \approx 0.25 - 0.20833 + 0.000757$$

$$\approx 0.0424$$
 (3)