King Saud University
Department of Mathematics
M-203[Final Badeel Examination]
(Differential and Integral Calculus)
(Summer-Semester 1438/39)

Max.Marks40 Time:3hrs

Marking Scheme: Q.No:1[2+3+4+3], Q.No2:[3+3+3+3], Q.No:3[4+4+4+4]

Q.No: 1 (a) Discuss the convergence of the sequence $\{\ln(2n) - \ln(2n-1)\}$.

(b) Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{1+3\sqrt{n}}$ is absolutely convergent, Conditional ly convergent, or divergent.

(c) Find the interval of convergence and radius of convergence of the power Series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n3^n}(x-1)^n$.

(d) Find Maclaurin's series for the function $f(x) = \frac{1}{(1+x)^2}$.

Q.No: 2 (a) Use double integral to find the volume of the region lying under the surface $z = 3$ and above the region bounded by the graphs of the equations $x + y = 2$, $2x + y = 4$, and $x = 0$.

(b) Find the area of the surface of the portion of paraboloid $z = x^2 + y^2$ that Is cut off by the plane $z = 1$.

(c) Use spherical coordinates to find the volume of the portion of the sphere $x^2 + y^2 + z^2 = 9$ that lies in the first octant.

(d) Use cylindrical coordinates to evaluate the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} dz\,dy\,dx$.

Q.No: 3 (a) Show that the integral $\int_C ye^{xy}dx + (xe^{xy} - 2y)dy$ is independent of path by finding the potential function.

(b) Use Green's theorem to find the integral Evaluate the integral $\oint_C \vec{F}\cdot\vec{dr}$, where

$$\oint_C \vec{F}\bullet\vec{dr} = \oint_C (y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy;$$

C is the boundary of the region enclosed by $x = y^2$ and $y = x^2$.

(c) Use Stokes's theorem to evaluate $\iint_S (curl\,\vec{F})\bullet\vec{n}dS$, , where $\vec{F}(x,y,z) = 2z\vec{i} + 3x\vec{j} + 5y\vec{k}$, and S is the surface of the region bounded by the portion of the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane.

(d) Find the flux integral of the vector field $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$ through the surface S, where S is the first octant portion of the plane $3x + 2y + z = 12$.