King Saud University
Department Of Mathematics
M-203 [Final Examination]
(Differential and Integral Calculus)
(Summer Semester 1435 1436)

Max. Marks: 40 Time: 3 hrs.

Marking Scheme: Q.No:1[3+5+4], Q.No:2[4+4+4], Q.No:3[3+3+4+6]

Q. No: 1 (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{\sin^{-1}(1/n)}{n^2}$ converges or diverges.

(b) Find the **interval of convergence** and **radius of convergence** of the power series $\sum_{n=0}^{\infty} \frac{n^2}{2^n}(x+1)^n$.

(c) Find Maclaurin series of $f(x) = \cos x$ and use it to find the Maclaurin series of $\sin^2 x$.

Q. No: 2 (a) Evaluate the integral $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$.

(b) Find the surface area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

(c) Sketch the graph of the solid region Q that lies inside the sphere $x^2 + y^2 + z^2 = 1$ and outside the cone $z^2 = x^2 + y^2$ and find its **volume** using spherical coordinates.

Q. No: 3 (a) Show that the line integral $\int_{(1,0,2)}^{(-2,1,3)} (6xy^3 + 2z^2)dx + 9x^2y^2dy + (4xz + 1)dz$ is independent of path, and find its value.

(b) Use the Green's theorem to evaluate

$\oint_C (\sqrt{x^2+1} - x^2 y)dx + (xy^2 - y^{\frac{5}{3}})dy$ where C is the circle $x^2 + y^2 = 4$.

(c) Use the Divergence theorem to find $\iint_S \vec{F}.\vec{n}$ dS, where

$\vec{F}(x,y,z) = x^3\vec{i} + y^3\vec{j} + 0\vec{k}$ and S is the surface of the region bounded by $z = 3 - x^2 - y^2$ and the plane z =1.

(d) Verify the Stoke's theorem for the vector field $\vec{F}$ and the surface S, where $\vec{F}(x,y,z) = 2z\vec{i} + 3x\vec{j} + 5y\vec{k}$ and S is the portion of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$ with upward orientation and C is the trace of S in the xy-plane.