King Saud University
Department Of Mathematics.
M-203 [Final Examination]
(Differential and Integral Calculus)
(II-Semester 1439/1440)

Max. Marks: 40 Time: 3 hrs

Marking Scheme:	**Q1[4+4+4];**	**Q2[4+4+4];**	**Q3[4+4+4+4].**

Q. No: 1 **(a)** Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{n^2-1}{2^n(n^2+5n)}$.

(b) Find the interval of convergence and radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{\ln^2(n)}{n}(x-1)^n.$$

(c) Find the MacLaurin series for $f(x) = \cos(x^3)$ and use its first three non-zero terms to approximate the integral $\int_0^1 \frac{1-\cos(x^3)}{x^6}\, dx$.

Q. No: 2 **(a)** A lamina has the shape of the region bounded by the graphs of the semi-circle $x = \sqrt{1-y^2}$ and the y-axis. If the density at a point $P(x, y)$ is directly proportional to the distance of P to the origin, find the moment of inertia about the y-axis.

(b) Find the centroid of the solid bounded by the graphs of the equations $z = 9 - x^2 - y^2$ and $z = 1 + x^2 + y^2$.

(c) Evaluate the triple integral by changing to spherical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} (x^2+y^2+z^2)^{\frac{3}{2}} dz dy dx.$$

Q. No: 3 **(a)** Using Green's theorem, calculate the line integral $\oint_C y^2 dx + (x+y)^2 dy$ where C is the triangle with vertices $A(1,0), B(1,1),$ and $D(0,1)$.

(b) Show that the following integral is independent of path and evaluate it:

$$\int_{(1,0,0)}^{(3,\frac{\pi}{2},1)} (2x \sin y + e^{3z})dx + (x^2 \cos y)dy + (3xe^{3z} + 5)dz$$

(c) Use the divergence theorem and cylindrical coordinates to find the Flux integral of the vector field $\vec{F}(x, y, z) = xy\,\vec{i} + yz\vec{j} + zx\vec{k}$ over the boundary S of the closed region Q bounded by the graphs of the equations $x^2 + y^2 = 1, z = 0,$ and $z = 2$. (Provided S is oriented by the unit normal directed outward).

(d) Use Stoke's theorem to evaluate the line integral $\oint_C \vec{F} \cdot \overrightarrow{dr}$ for the vector field $\vec{F}(x, y, z) = (y - z)\,\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}$, C is the boundary of the part of the plane $2x + 3y + z = 6$ in the first octant oriented in a counterclockwise direction when viewed from above.