M-203- Final Examination (1438-1439), Semester-I
Department of Mathematics, College of Science
King Saud University

Max. Marks-40 Time-3 Hours

Attempt all questions

Question1: (i) Determine whether the sequence

$$\left\{\frac{\cos\frac{1}{n}-1}{\sin\frac{1}{n}}\right\},$$

converges or diverges and if it converges, find its limit. (2)

(ii) Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{e^n}{\sqrt{e^{n^2}}}$$

is absolutely convergent, conditionally convergent or divergent. (3)

(iii) Find the interval of convergence and radius of convergence of the power series (4)

$$\sum_{n=2}^{\infty}(-1)^n\frac{(3x+5)^n}{n\ln n}$$

(iv) Find the Maclaurin series representation of the function $f(x)=\tan^{-1}x$, $|x|\leq 1$ (3)

and use its first three terms to approximate the integral

$$\int_0^{0.5}\tan^{-1}x\,dx.$$

Question2: (i) Evaluate the double integral (3)

$$\int_0^{\sqrt{2}}\int_{y^2}^{2}y^3e^{x^3}\,dx\,dy.$$

(ii) Find the surface area of the portion of the hemisphere $z=\sqrt{4-x^2-y^2}$ lying inside the cylinder $x^2+y^2=1$ in the first octant. (3)

(iii) Evaluate the integral

$$\iiint\limits_{Q} (x^2 + y^2) dv,$$

where the solid Q is bounded by the cone $z = \sqrt{x^2 + y^2}$ and $z = 2$. (3)

(iv) Evaluate the integral by changing to spherical coordinates (3)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2 + y^2 + z^2) dz dy dx$$

Question 3. (i) Let $\overrightarrow{F} = (2x + y)i + (2y + x)j$. Show that the line integral

$$\int_c \overrightarrow{F}.d\overrightarrow{r}$$

is independent of path by finding the potential function of $\overrightarrow{F}$. (4)

(ii) Use Green's theorem to evaluate the line integral

$$\oint_c (y^2 - x^2)\, dx + (x^2 + y^2) dy,$$

where c is the boundary of the triangle: $x = 3$, $y = 0$ and $y = x$. (4)

(iii) Use divergence theorem to find the flux of the force $\overrightarrow{F} = (x^3 - yz)\, i + (y^3 + zx)\, j + (z^3 - xy)k$ through the surface bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1$. (4)

(iv) Verify Stokes theorem for $\overrightarrow{F} = yzi + zxj + xyk$ and S, where S is the surface of the paraboloid $z = x^2 + y^2$ cut off by the plane $z = 1$. (4)