

King Saud University
Department Of Mathematics.
M-203 [Final Examination]
(Differential and Integral Calculus)
 (I-Semester 1437/1438)

Max. Marks: 40

Time: 3 hrs

Marking Scheme: Q.1[2+3]; Q.2[4+3]; Q.3[3+3]; Q.4[3+3]; Q.5[4]; Q.6[4]; Q.7[4]; Q.8[4].

Q. No: 1 (a) Discuss the convergence of the sequence $\left\{ \left(\frac{n-3}{n} \right)^n \right\}_{n=1}^{\infty}$

(b) Find the sum of the series: $\sum_{n=1}^{\infty} \left[\frac{1}{(n+3)(n+4)} + \frac{1}{2^n} \right]$

Q. No: 2 (a) Find the interval of convergence and radius of convergence of the

power series $\sum_{n=1}^{\infty} (-1)^n \frac{2^n x^n}{\sqrt{1+n}}$.

(b) Use the first three non-zero terms of the Maclaurin series to approximate the

integral $\int_0^{0.5} e^{-x^2} dx$, to four decimal places.

Q. No: 3 (a) Use a double integral to find the volume of the solid bounded by the plane $z = 4 - 4x - 2y$ and the coordinate planes.

(b) Evaluate the integral $\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3) dy dx$.

Q. No:4 (a) Use cylindrical coordinates to evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2) dz dy dx$.

(b) Find the volume of the region that lies between two concentric spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 4$.

Q. No: 5 Find the potential function of the vector field

$\vec{F}(x, y, z) = (2xy - z^2)\vec{i} + (x^2 + 2z)\vec{j} + (2z + 2y - 2xz)\vec{k}$ and evaluate

the integral $\int_{(1,2,3)}^{(3,2,1)} \vec{F} \cdot d\vec{r}$.

Q. No: 6 Use Green's theorem to evaluate $\oint_C 2xy dx + xy^2 dy$ where C is the triangle with vertices (0,0), (1,1), and (2,0).

Q. No: 7 Use divergence theorem to evaluate $\iiint_V \vec{F} \cdot \vec{n} \, dS$ where

$\vec{F}(x, y, z) = y^3\vec{i} + z\vec{j} + z\vec{k}$ and S is the surface bounded by the graphs of $z = 1 - x^2 - y^2$ and $z = 0$.

Q. No:8 Use Stokes's theorem to evaluate $\iint_S \text{curl}(\vec{F}) \cdot \vec{n} \, dS$, where

$\vec{F}(x, y, z) = y\vec{i} - x\vec{j} - z^2\vec{k}$ and S is the part of surface bounded by $z = 4 - x^2 - y^2$ and $z = 0$ oriented upward.

(1)

M-203 I Sem (1437/1438) (1)

Q.1 (a)

$$\left\{ \left(\frac{n-3}{n} \right)^n \right\}_{n=1}^{\infty}$$

[Mark: 2]

$$a_n = \left(1 - \frac{3}{n} \right)^n$$

1^{∞} form (1)

$$\ln a_n = n \ln \left(1 - \frac{3}{n} \right)$$

(1) $\infty \cdot 0$ - form

$$= \frac{\ln \left(1 - \frac{3}{n} \right)}{\frac{1}{n}}$$

$\frac{0}{0}$ - form

$$\lim_{x \rightarrow \infty} \ln a_n = \lim_{x \rightarrow \infty} \frac{\frac{1}{1-3/x} \left(\frac{3}{x^2} \right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{-3}{1-3/x} = -3$$

$$\lim_{x \rightarrow \infty} a_n = e^{-3} \quad (1) \quad \#$$

$$1(b) \sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)} + \sum \frac{1}{2^n} = \sum_{n=1}^{\infty} \left[\frac{1}{n+3} - \frac{1}{n+4} \right] + \sum_{n=1}^{\infty} \frac{1}{2^n}$$

[Mark: 3]

$$s_n = \left[\frac{1}{4} - \frac{1}{5} \right] + \left[\frac{1}{5} - \frac{1}{6} \right] + \dots + \left[\frac{1}{n+3} - \frac{1}{n+4} \right]$$

$$s_n = \frac{1}{4} - \frac{1}{n+4} \Rightarrow \lim s_n = \frac{1}{4} \quad (2)$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \text{ G.S.} \Rightarrow S = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

(1)

$$s_{\infty} = \frac{1}{4} + 1 = \frac{5}{4} \quad \#$$

$$Q.2 (a) e^x \approx 1 + x + \frac{x^2}{2!} \Rightarrow e^{-x^2} \approx 1 - x^2 + \frac{x^4}{2!} \quad [\text{Mark: 3}]$$

$$\int_0^{0.5} [1 - x^2 + \frac{x^4}{2}] dx = \left[x - \frac{x^3}{3} + \frac{x^5}{10} \right]_0^{0.5} = (0.5) - \frac{(0.5)^3}{3} + \frac{(0.5)^5}{10} \approx 0.4615 \quad (1)$$

$$2(a) \left| \frac{u_{n+1}}{u_n} \right| = \frac{2^{n+1}}{\sqrt{n+2}} \times \frac{\sqrt{n+1}}{2^n} |x|$$

[Mark: 4]

$$\text{c'st } 2|x| < 1 \quad -\frac{1}{2} < x \leq \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = 2|x| \quad (1)$$

$$\text{At } x = -\frac{1}{2} \text{ dist } (1) \quad \text{At } x = \frac{1}{2} \text{ c'st } (1) \quad (1) \quad P = \frac{1}{2}$$

(2)

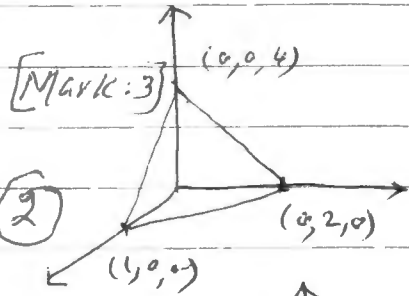
Q3 (a)

$$V = \int_0^1 \int_0^{2-2x} (4-4x-2y) dy dx$$

$$= \int_0^1 [4y - 4xy - y^2]_0^{2-2x} dx$$

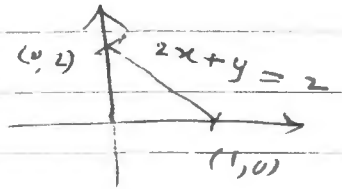
$$= \int_0^1 [4 - 8x + 4x^2] dx$$

$$= [4x - 4x^2 + \frac{4x^3}{3}]_0^1 = 4 - 4 + \frac{4}{3} = \frac{4}{3} \text{ (1) \#}$$



[Mark: 3]

(2)



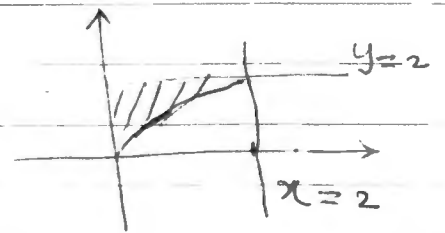
(b) $\int_0^2 \int_{\sqrt{x}}^2 \sin y^3 dy dx$ [Mark: 3]

$$= \int_0^2 \int_0^y \sin y^3 dx dy \text{ (2)}$$

$$= \int_0^2 \sin(y^3) y^2 dy = \frac{1}{3} \int_0^2 \sin(y^3) (3y^2) dy$$

$$= \frac{1}{3} [\cos(y^3)]_0^2 = \frac{1}{3} \cos(8) - \frac{1}{3}$$

$$= \frac{1}{3} [\cos 8 - 1] \text{ (1) \#}$$



$$0 \leq y \leq 2$$

$$0 \leq x \leq y^2$$

Q4 (a)

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2) dz dy dx = \int_0^{\pi/2} \int_0^{\sqrt{1-r^2}} (r^2) r dr d\theta$$

$$= \int_0^{\pi/2} \int_0^1 r^3 \sqrt{1-r^2} dr d\theta$$

[Mark: 3]

$$r = \sin \theta, dr = \cos \theta d\theta$$

$$\int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta = \dots$$

$$u = \cos \theta, du = -\sin \theta d\theta$$

$$= - \int_0^{\pi/2} (1 - \cos^2 \theta) \cos^2 \theta (-\sin \theta) d\theta$$

$$= - \int_1^0 (1 - u^2) u^2 du = - \int_0^1 (u^2 - u^4) du$$

$$= - \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_0^1 \text{ \#}$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15} \text{ (1) \#}$$

(1) \#

(3)

Q: 4(b) Using spherical coordinates [Mark: 3]

$$V = \int_0^{2\pi} \int_0^{\pi} \int_2^3 \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta = \int_0^{2\pi} \int_0^{\pi} \left[\frac{\rho^3}{3} \right]_2^3 \sin \varphi \, d\varphi \, d\theta$$

$$= \left(\frac{27}{3} - \frac{8}{3} \right) \int_0^{2\pi} \left[-\cos \varphi \right]_0^{\pi} d\theta = \frac{19}{3} (2)(2\pi) = \frac{76}{3} \pi$$

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Q: 5(a) $\vec{F} = \nabla f$ [Mark: 4]

$$f_x = 2xy - z^2 \Rightarrow f(x, y, z) = x^2 y - z^2 x + c_1$$

$$f_y = x^2 + 2z \Rightarrow f(x, y, z) = x^2 y + 2zy + c_2$$

$$f_z = 2z + 2y - 2xz \Rightarrow f(x, y, z) = z^2 + 2yz - xz^2 + c_3$$

$$\Rightarrow f(x, y, z) = x^2 y - z^2 x + 2zy + z^2 + c$$

$$\left[f(x, y, z) \right]_{(1, 2, 3)}^{(3, 3, 1)} = [18 - 3 + 4 + 1] - [2 - 9 + 12 + 9]$$

(1, 2, 3)

$$= 20 - 14 = 6$$

(1)

Q: 6(f) $\iiint (0 + 0 + 1) \, dv$ [Mark: 4] $0 \leq z \leq 1 - r^2$

Q

$$= \int_0^{2\pi} \int_0^1 \int_0^{1-r^2} r \, dz \, dr \, d\theta$$

$$0 \leq r \leq 1$$

$$0 \leq \theta \leq 2\pi$$

$$= \int_0^{2\pi} \int_0^1 r(1-r^2) \, dr \, d\theta = \int_0^{2\pi} \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 d\theta = \frac{1}{2} (2\pi) = \pi$$

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(4)

$$Q: 76) \oint_C 2xy \, dx + xy^2 \, dy = \iint_{R_{xy}} (y^2 - 2x) \, dA \quad [\text{Mark: 4}]$$

$$= \int_0^1 \int_y^{-y+2} (y^2 - 2x) \, dx \, dy \quad (2)$$

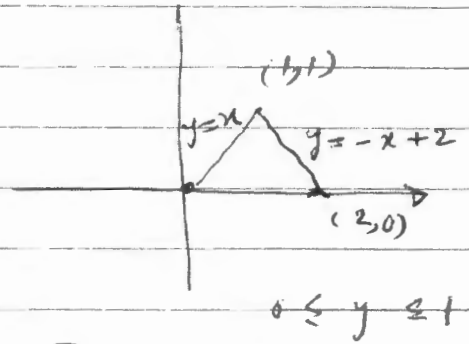
$$= \int_0^1 \left[y^2 x - x^2 \right]_y^{-y+2} dy$$

$$= \int_0^1 \left[y^2(-y+2) - (-y+2)^2 \right] - \left[y^3 - y^2 \right] dy \quad y \leq x \leq -y+2$$

$$= \int_0^1 \left[-y^3 + 2y^2 - y^2 + 4y - 4 - y^3 + y^2 \right] dy$$

$$= \int_0^1 \left[-2y^3 + 2y^2 + 4y - 4 \right] dy = \left[-\frac{y^4}{2} + \frac{2}{3}y^3 + 2y^2 - 4y \right]_0^1$$

$$= \left[-\frac{1}{2} + \frac{2}{3} + 2 - 4 \right] = -\frac{11}{6} \quad (2)$$



$$Q: 8 \quad \oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C y \, dx - x \, dy - z^2 \, dz \quad [\text{Mark: 4}]$$

$$x = 2 \cos \theta, \quad y = 2 \sin \theta, \quad z = 0 \quad 0 \leq \theta \leq 2\pi$$

$$= \int_0^{2\pi} (2 \sin \theta)(-2 \sin \theta) - 2 \cos \theta (2 \cos \theta) \, d\theta \quad (2)$$

$$= -4 \int_0^{2\pi} (\sin^2 \theta + \cos^2 \theta) \, d\theta = -4(2\pi) = -8\pi \quad (1)$$

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