KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1430/1431)

Correction Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice(1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No.	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	**c**	**b**	**b**	**b**	**a**	**a**	**c**	**a**	**a**	**c**

Q. No: 1 The partial fraction decomposition of $\frac{1}{x^4-1}$ takes the form

(a) $\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x^2+1}$ (b) $\frac{A}{x^2-1}+\frac{Bx+C}{x^2+1}$ (c) $\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}$ (d) None of these

Q. No: 2 To evaluate the integral $\int \cos^3(x)dx$, we substitute

(a) $u=\cos x$, (b) $u=\sin x$, (c) $u=\cos^2(x)$, (d) None of these

Q. No: 3 The integral $\int \frac{1}{\sqrt{x^2+2x+5}}dx$ is equal to

(a) $\sinh^{-1}\left(\frac{x+1}{4}\right)+c$, (b) $\sinh^{-1}\left(\frac{x+1}{2}\right)+c$ (c) $\frac{1}{2}\sinh^{-1}\left(\frac{x+1}{2}\right)+c$, (d) None of these

Q. No: 4 $\lim_{x\to\infty}\frac{e^x+e^{2x}}{1+e^{2x}}$ is equal to

(a) ∞, (b) 1, (c) 0, (d) None of these

Q. No: 5 If $f(x)=\cosh^{-1}\left(\sqrt{x}\right)$, then $f'(x)$ is equal to

(a) $\frac{1}{2\sqrt{x^2-x}}$, (b) $\frac{1}{2\sqrt{x-x^2}}$, (c) $\frac{1}{2\sqrt{x^2+x}}$, (d) None of these

Q. No: 6 The improper integral $\int_0^\infty \frac{1}{4+x^2}dx$

(a) Converges to $\frac{\pi}{4}$, (b) Converges to $\frac{\pi}{2}$, (c) diverges , (d)None of these

Q. No: 7 The integral $\int x\sin(2x)dx$ is equal to

(a) $\frac{x^2}{2}\cos(2x)+c$, (b) $\frac{x}{2}\cos(2x)-\frac{1}{2}\sin(2x)+c$, (c) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+c$,

(d) None of these

Q. No: 8 The value of the integral $\int_3^8 \frac{\sqrt{1+x}}{x}dx$ is equal to

(a) $2+\ln\left(\frac{3}{2}\right)$, (b) $2-\ln 2$, (c) $2+\ln 2-\ln 3$, (d) None of these

Q. No: 9 The integral $\int\sqrt{1+\sqrt{x}}dx$ is equal to

(a) $\frac{4}{5}\left(1+\sqrt{x}\right)^{\frac{5}{2}}-\frac{4}{3}\left(1+\sqrt{x}\right)^{\frac{3}{2}}+c$, (b) $\frac{4}{5}\left(1+\sqrt{x}\right)^{\frac{5}{2}}+c$, (c) $\frac{4}{3}\left(1+\sqrt{x}\right)^{\frac{3}{2}}+c$, (d) None of these.

Q. No: 10 The integral $\int \frac{x-2}{x\sqrt{x^2-25}}dx$ is equal to

(a) $\cosh^{-1}\left(\frac{x}{5}\right)-2\sec^{-1}\left(\frac{x}{5}\right)+c$, (b) $\cosh^{-1}\left(\frac{x}{5}\right)-\frac{2}{5}\sec^{-1}(x)+c$, (c) $\cosh^{-1}\left(\frac{x}{5}\right)-\frac{2}{5}\sec^{-1}\left(\frac{x}{5}\right)+c$,

(d) None of these

Full Questions

Question No: 11 Evaluate $\int \sin^3(x)\cos^7(x)dx$ [2]

Let $u=\cos x \Rightarrow du=-\sin x dx$ (0.5)

So

$$\int \sin^3(x)\cos^7(x)dx=\int u^7\left(u^2-1\right)du \quad (0.5)$$

$$=\frac{1}{10}u^{10}-\frac{1}{8}u^8+c$$

$$=\frac{1}{10}(\cos x)^{10}-\frac{1}{8}(\cos x)^8+c \quad (1)$$

Question No: 12 Evaluate $\int \tan^3(x)\sec^6(x)dx$ [2]

Let $u=\sec x \Rightarrow du=\sec x\tan x dx$ (0.5)

So

$$\int \tan^3(x)\sec^6(x)dx=\int u^5\left(u^2-1\right)du \quad (0.5)$$

$$=\frac{1}{8}u^8-\frac{1}{6}u^6+c$$

$$=\frac{1}{8}(\sec x)^8-\frac{1}{6}(\sec x)^6+c \quad (1)$$

Question No: 13 Evaluate $\int \frac{1}{(x-1)(x^2+1)}dx$. [3]

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2}\frac{1}{(x-1)} - \frac{1}{2}\frac{x+1}{(x^2+1)} \qquad (1)$$

So

$$\int \frac{1}{(x-1)(x^2+1)}dx = \frac{1}{2}\int\left(\frac{1}{(x-1)} - \frac{2x}{2(x^2+1)} - \frac{1}{(x^2+1)}\right)dx \qquad (0.5)$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + c \qquad (0.5+0.5+0.5)$$

Question No: 14 Evaluate $\int (1-x^2)^{\frac{3}{2}}dx$. [3]

Let $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$ (0.5)

So

$$\int (1-x^2)^{\frac{3}{2}}dx = \int \cos^4\theta d\theta \qquad (0.5)$$

$$= \int\left(\frac{1+\cos(2\theta)}{2}\right)^2 d\theta \qquad (0.5)$$

$$= \frac{1}{8}\int (3 + 4\cos(2\theta) + \cos(4\theta))\, d\theta$$

$$= \frac{1}{8}\left(3\theta + 2\sin(2\theta) + \frac{1}{4}\sin(4\theta)\right) \qquad (0.5)$$

$$= \frac{1}{8}\left(3\sin^{-1}(x) + 2\sin(2\sin^{-1}(x)) + \frac{1}{4}\sin(4\sin^{-1}(x))\right) + c$$

$$= \frac{1}{8}\left(3\sin^{-1}(x) + 4x\sqrt{1-x^2} + x\sqrt{1-x^2}(1-2x^2)\right) + c. \qquad (1)$$