



KING SAUD UNIVERSITY  
*College of Science*  
*Department of Mathematics*

# M-106

First Semester (1431/1432)

Solution Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: Three hours

Marks:

Multiple Choice (1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

## Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\{a, b, c, d\}$	$d$	$d$	$c$	$a$	$d$	$b$	$b$	$d$	$b$	$a$	$c$	$c$	$d$	$a$	$a$	$b$	$a$	$d$	$a$	$d$

Q. No: 1 If  $\sum_{k=3}^5 (k + \alpha) = 6$ , then the value of  $\alpha$  is equal to:

- (a)  $-\frac{9}{5}$                       (b)  $\frac{9}{5}$                       (c) 2                      (d) -2

Q. No: 2 The average value of the function  $f(x) = \sin x \cos x$  on  $[-\pi, \pi]$  is equal to:

- (a)  $\frac{1}{2\pi}$                       (b)  $\frac{1}{\pi}$                       (c) 1                      (d) 0

Q. No: 3 The integral  $\int \tan(2x) dx$  is equal to:

- (a)  $\frac{-1}{2} \ln |\sec(2x)| + c$     (b)  $\frac{1}{2} \sec^2(2x) + c$     (c)  $\frac{-1}{2} \ln |\cos(2x)| + c$     (d)  $2 \sec^2(2x) + c$

Q. No: 4 If  $F(x) = \int_1^{x^2} \ln(t^2) dt$ , then  $F'(2)$  is equal to:

- (a)  $4 \ln 16$                       (b)  $2 \ln 16$                       (c)  $4 \ln 8$                       (d)  $2 \ln 8$

Q. No: 5 The integral  $\int \ln(2^{\sin x}) dx$  is equal to:

- (a)  $\frac{1}{2} \ln(2) \sin x + c$     (b)  $2^{-\sin x} \cos x + c$     (c)  $-\sin x + c$     (d)  $-(\cos x) \ln 2 + c$

Q. No: 6 If  $f(x) = \tanh^{-1}(\cos(3x))$  then  $f'(x)$  is equal to:

- (a)  $3 \csc(3x)$                       (b)  $-3 \csc(3x)$                       (c)  $\frac{-3 \sin(3x)}{1 + \cos^2(3x)}$                       (d) 0

Q. No: 7  $\lim_{x \rightarrow 1} \left( \frac{1}{x-1} - \frac{1}{\ln x} \right)$  is equal to:

- (a)  $\infty$                       (b)  $-\frac{1}{2}$                       (c)  $\frac{1}{2}$                       (d) 0

Q. No: 8 To evaluate the integral  $\int \frac{1}{\sqrt[3]{x-\sqrt{x}}} dx$ , we use the substitution:

- (a)  $u = \sqrt{x}$                       (b)  $u = \sqrt[4]{x}$                       (c)  $x = \sqrt[6]{u}$                       (d)  $u = \sqrt[6]{x}$

Q. No: 9 The integral  $\int \frac{\cos x}{1 + \sin^2(x)} dx$  is equal to:

- (a)  $\frac{1}{1 + \sin x} + c$     (b)  $\tan^{-1}(\sin x) + c$     (c)  $\frac{1}{1 + \cos x} + c$     (d)  $\tanh^{-1}(\sin x) + c$

Q. No: 10 To evaluate the integral  $\int \frac{1}{x^4 \sqrt{x^2-7}} dx$ , we use the substitution:

- (a)  $x = \sqrt{7} \sec \theta$     (b)  $x = 7 \sec \theta$     (c)  $x = 7 \tan \theta$     (d)  $x = \sqrt{7} \tan \theta$

Q. No: 11 The partial fraction decomposition of  $\frac{x-1}{x^2(x^2+1)}$  takes the form:

(a)  $\frac{A}{x} + \frac{Bx+C}{(x^2+1)}$  (b)  $\frac{A}{x} + \frac{B}{(x^2+1)}$  (c)  $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$  (d)  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+1}$

Q. No: 12 The improper integral  $\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}} dx$

(a) converges to  $-2$  (b) converges to  $1$  (c) converges to  $2$  (d) Diverges

Q. No: 13 The area of the region bounded by the graphs of the functions  $y = x^2$ ,  $y = 2-x^2$  is equal to:

(a)  $2$  (b)  $4$  (c)  $\frac{3}{8}$  (d)  $\frac{8}{3}$

Q. No: 14 The slope of the tangent line at the point corresponding to  $t = 1$  on the curve given parametrically by the equations  $x = 2t^3 + 1$ ,  $y = 5t^3 - 1$ ,  $-2 \leq t \leq 2$  is:

(a)  $\frac{5}{2}$  (b)  $-\frac{5}{2}$  (c)  $\frac{2}{5}$  (d)  $-\frac{2}{5}$

Q. No: 15 If a graph has polar equation  $r = 2 \sec \theta$ , then its equation in  $xy$ -system is:

(a)  $x = 2$  (b)  $y = 2$  (c)  $x + y + 1 = 0$  (d)  $y = \frac{1}{2}$

Q. No: 16 The length of the curve  $C : x = \cos(2t)$ ,  $y = \sin(2t)$ ,  $0 \leq t \leq \pi$  is equal to:

(a)  $2$  (b)  $2\pi$  (c)  $\pi$  (d)  $4\pi$

Q. No: 17 The surface area resulting by revolving the graph of the parametric equation  $x = 3t$ ,  $y = 3t$ ,  $0 \leq t \leq 1$  around the  $x$ -axis is equal to:

(a)  $9\sqrt{2}\pi$  (b)  $18\sqrt{2}\pi$  (c)  $24\sqrt{2}\pi$  (d)  $\frac{9}{2}\sqrt{2}\pi$

Q. No: 18 If a point has  $xy$ -coordinates  $(x, y) = (1, 1)$  then one of its  $(r, \theta)$ -coordinates is:

(a)  $\left(1, \frac{\pi}{2}\right)$  (b)  $\left(-1, \frac{5\pi}{4}\right)$  (c)  $\left(2, \frac{\pi}{4}\right)$  (d)  $\left(\sqrt{2}, \frac{\pi}{4}\right)$

Q. No: 19 The slope of the tangent line to the graph of the equation  $r = 2$  at  $\theta = -\frac{\pi}{4}$  is:

(a)  $1$  (b)  $-1$  (c)  $0$  (d)  $\infty$

Q. No: 20 The graph of the curve  $\mathcal{C}$  defined by the parametric equations  $x = 2 + \cos(2t)$ ;  $y = -1 + \sin(2t)$ ,  $0 \leq t \leq \pi$  is a:

(a) line (b) parabola (c) cardioids (d) circle

## Full Questions

Question No: 21 Approximate the integral  $\int_0^4 \sqrt{x^3 + 1} dx$  using the **Simpson's rule** with  $n = 4$ . [4]

**Solution:**

Let  $f(x) = \sqrt{x^3 + 1}$ .

$$\Delta x = \frac{4}{4} = 1$$

$$x_0 = 0, \quad x_1 = 1, \quad x_2 = 2, \quad x_3 = 3 \quad \text{and} \quad x_4 = 4. \quad (1)$$

$$\begin{aligned} \int_0^4 \sqrt{x^3 + 1} dx &\approx \frac{4-0}{3 \times 4} \{f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\} \quad (2) \\ &= \frac{1}{3} \{1 + 4(\sqrt{2}) + 2(3) + 4(\sqrt{28}) + \sqrt{65}\} \quad (1) \\ &= \frac{1}{3} \{1 + 5.6568 + 6 + 21.166 + 8.0623\} \\ &= \frac{1}{3} \{41.885\} \approx 13.962 \end{aligned}$$

Question No: 22 Evaluate  $\int \frac{1}{\sqrt{x^2 + 4x + 5}} dx$ . [4]

**Solution:**

$$\begin{aligned} \int \frac{1}{\sqrt{x^2 + 4x + 5}} dx &= \int \frac{1}{\sqrt{(x+2)^2 + 1}} dx \quad (1) \\ &= \int \frac{1}{\sqrt{u^2 + 1}} du \quad (\text{with } u = x + 2) \quad (2) \\ &= \sinh^{-1}(u) + c \\ &= \sinh^{-1}(x + 2) + c \quad (1) \end{aligned}$$

Question No: 23 **Evaluate**  $\int \frac{1}{(x^2 + 9)^2} dx$  [6]

**Solution:**

$$\text{Let } x = 3 \tan \theta \Rightarrow dx = 3 \sec^2 \theta d\theta \quad (1)$$

$$\int \frac{1}{(x^2 + 9)^2} dx = \int \frac{3 \sec^2 \theta}{(9 \tan^2 \theta + 9)^2} d\theta$$
$$= \frac{1}{27} \int \frac{1}{\sec^2 \theta} d\theta \quad (1)$$

$$= \frac{1}{27} \int \cos^2 \theta d\theta$$
$$= \frac{1}{27} \int \left( \frac{1 + \cos(2\theta)}{2} \right) d\theta \quad (1)$$

$$= \frac{1}{54} (\theta + \sin \theta \cos \theta) + c \quad (1)$$

$$= \frac{1}{54} \left( \tan^{-1} \left( \frac{x}{3} \right) + \frac{3x}{x^2 + 9} \right) + c \quad (2)$$

Question No: 24 **Evaluate**  $\int \frac{\sin^{-1}(\ln x)}{x} dx$  [5]

**Solution:**

$$\text{Let } u = \ln x \Rightarrow du = \frac{1}{x} dx \quad (1)$$

$$\int \frac{\sin^{-1}(\ln x)}{x} dx = \int \sin^{-1}(u) du \quad (1)$$

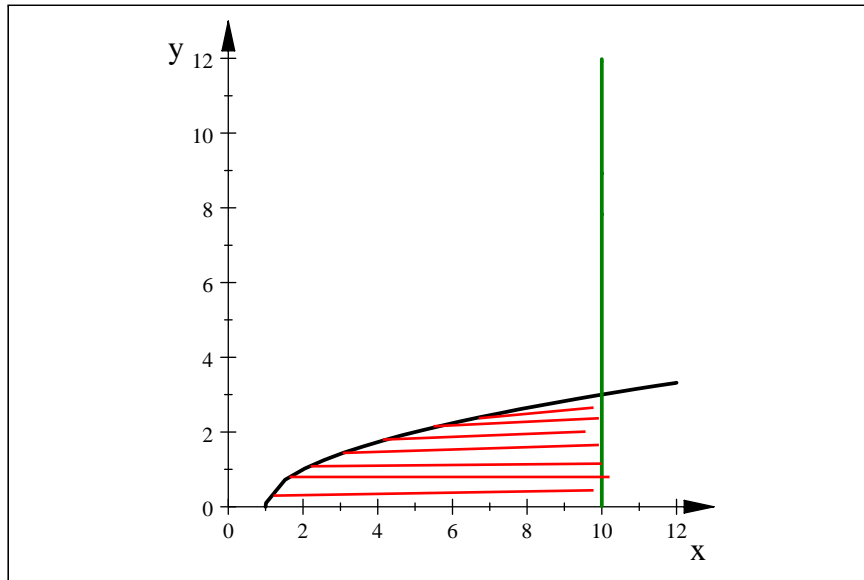
$$= u \sin^{-1} u + \sqrt{1 - u^2} + c \quad (2)$$

$$= (\ln x) \sin^{-1}(\ln x) + \sqrt{1 - (\ln x)^2} + c \quad (1)$$

Question No: 25 **Sketch** the region  $R$  bounded by the graph of the equations  $y = \sqrt{x-1}$ ,  $x = 10$  and  $y = 0$ . **Find** the **volume** of the solid generated by revolving the region  $R$  around the  $y$ -axis. (Use any method) [6]

**Solution:**

Graph: (3)



By Washer Method:

$$V = \pi \int_0^3 \left( (10)^2 - (y^2 + 1)^2 \right) dy = \frac{1152}{5} \pi. \quad (2 + 1)$$

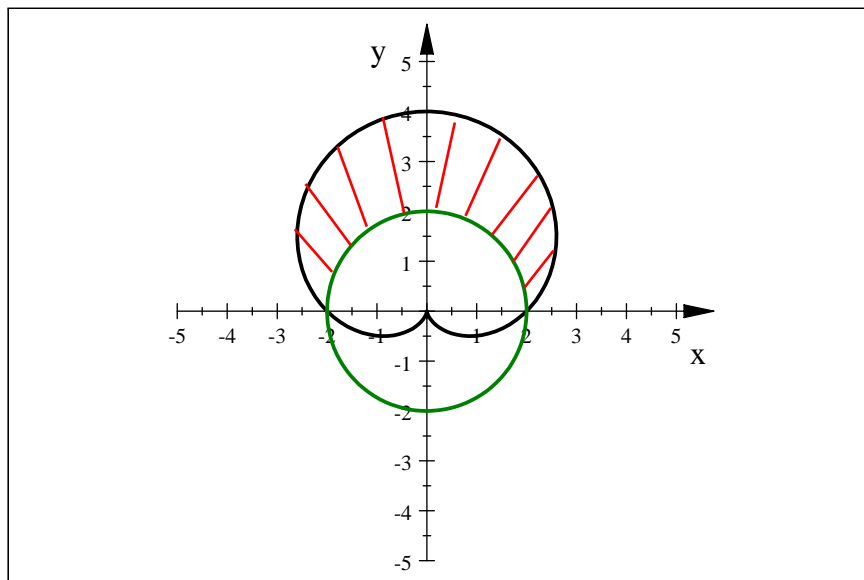
By Cylindrical shell method:

$$V = 2\pi \int_1^{10} x\sqrt{x-1} dx = \frac{1152}{5} \pi. \quad (2 + 1)$$

Question No: 26 **Sketch** the region  $R$  that lies inside the graph the equation  $r = 2 + 2 \sin \theta$  and outside the graph of the equation  $r = 2$  and **set up** (Do not evaluate) an integral that can be used to find its **area**. [5]

**Solution:**

Graph (3)



$$A = \frac{1}{2} \int_0^{\pi} ((2 + 2 \sin \theta)^2 - 2^2) d\theta \quad (2)$$