KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1431/1432)

Solution Second Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a, b, c, d\}$	a	b	d	c	d	b	b	a	d	a

Q. No: 1 $\lim_{x\to 0^+} x^x$ is equal to:

(a) 1 (b) 0 (c) ∞ (d) -∞

Q. No: 2 The partial fraction decomposition of $\dfrac{3x^2+x}{(x-2)(x^2+3)}$ takes the form:

(a) $\frac{A}{x-2}+\frac{B}{x^2+3}$ (b) $\frac{A}{x-2}+\frac{Bx+C}{x^2+3}$ (c) $\frac{A}{x-2}+\frac{Bx}{x^2+3}$ (d) $\frac{A}{x-1}+\frac{B}{x+3}+\frac{C}{x-3}$

Q. No: 3 To evaluate the integral $\displaystyle\int x^3\sqrt{x^2+8}dx$, we use the substitution:

(a) $x = 2\sec\theta$ (b) $x = 2\sqrt{2}\cos\theta$ (c) $x = 2\sqrt{2}\sec\theta$ (d) $x = 2\sqrt{2}\tan\theta$

Q. No: 4 The value of the integral $\displaystyle\int_0^{\frac{\pi}{4}} \cos(x)\sin^3(x)\,dx$ is equal to:

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{1}{16}$ (d) $\frac{3}{7}$

Q. No: 5 The substitution $u = \tan\left(\frac{x}{2}\right)$ transforms the integral $\displaystyle\int \frac{1}{\sin x}dx$ into:

(a) $\int \frac{-1}{u}du$ (b) $\int \frac{1}{u^2+1}du$ (c) $\int \frac{2}{u}du$ (d) $\int \frac{1}{u}du$

Q. No: 6 If $\displaystyle\int \sqrt{\tan x}\sec^2 x dx = \int 2u^2\,du$ then

(a) $u = \tan x$ (b) $u = \sqrt{\tan x}$ (c) $u = \sec^2 x$ (d) $u = \sec x$

Q. No: 7 The improper integral $\displaystyle\int_0^{\infty} \frac{1}{x+1}dx$

(a) converges to 0 (b) diverges (c) converges to 2 (d) converges to 1

Q. No: 8 The value of the integral $\displaystyle\int \frac{1}{\sqrt{-x^2+10x-21}}dx$ is equal to:

(a) $\sin^{-1}\left(\frac{x-5}{2}\right)+c$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{x-5}{2}\right)+c$ (c) $\sinh^{-1}\left(\frac{x-5}{2}\right)+c$

(d) $\sin^{-1}(x-5)+c$

Q. No: 9 The area of the region **bounded** by the graphs of equations: $y = e^{2x}$, $y = e^x$, $x = 0$ and $x = \ln 4$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{2}{9}$ (c) 2 (d) $\frac{9}{2}$

Q. No: 10 The value of the integral $\displaystyle\int \tan^2 x \sec^4 x dx$ is equal to:

(a) $\frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$ (b) $-\frac{1}{3}\tan^3 x - \frac{1}{5}\tan^5 x + c$

(c) $\frac{1}{3}\tan^3 x - \frac{1}{5}\tan^5 x + c$ (d) $-\frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$

Full Questions

Question No: 11 **Evaluate** $\displaystyle\int 2x \sec^{-1}(x)\, dx$ [**2**]

Solution:

$$\text{Let} \begin{cases} u = \sec^{-1} x \\ v' = 2x \end{cases} \Longrightarrow \begin{cases} u' = \frac{1}{x\sqrt{x^2-1}} \\ v = x^2 \end{cases} \quad (1)$$

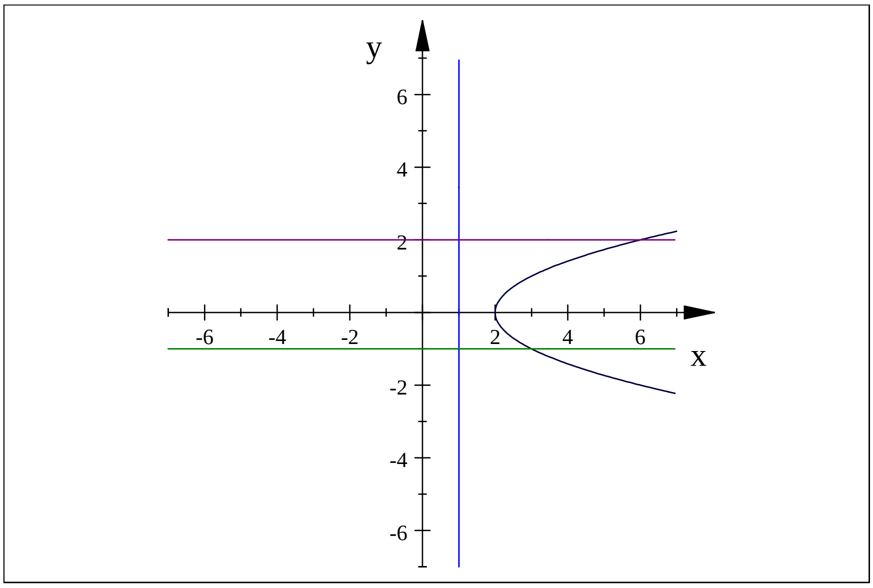
So

$$\int 2x \sec^{-1}(x)\, dx = x^2 \sec^{-1} x - \int \frac{x}{\sqrt{x^2-1}} dx \quad (0.5)$$

$$= x^2 \sec^{-1} x - \sqrt{x^2-1} + c \quad (0.5)$$

Question No: 12 Let R be the region **bounded** by the graphs of $x = y^2 + 2$, $x = 1$, $y = -1$ and $y = 2$. **Sketch** the region R and **find** its area. [**2**]

Solution: Graph (1)



Area: $A = \int_{-1}^{2} \left((y^2+2)-1\right) dy = 6.$ (0.5 + 0.5)

Question No: 13 **Evaluate** $\int \frac{x^2}{\sqrt{9-x^2}} dx$ [**3**]

Solution:

Let $x = 3\sin\theta$, $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$. $(dx = 3\cos\theta d\theta)$ (0.5)

$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = 3\cos\theta.$

So

$$\int \frac{x^2}{\sqrt{9-x^2}} dx = 9\int \sin^2\theta d\theta \qquad (0.5)$$

$$= \tfrac{9}{2}\int (1-\cos(2\theta))\, d\theta \qquad (0.5)$$

$$= \tfrac{9}{2}\theta - \tfrac{9}{2}\sin\theta\cos\theta + c \qquad (0.5)$$

$$= \tfrac{9}{2}\sin^{-1}\left(\tfrac{x}{3}\right) - \tfrac{1}{2}x\sqrt{9-x^2} + c \qquad (1)$$

Question No: 14 **Evaluate** $\int \frac{x-2}{x^3+x} dx$ [**3**]

Solution:

$$\frac{x-2}{x^3+x} = -\frac{2}{x} + \frac{2x+1}{x^2+1} \qquad (0.5+0.5+0.5)$$

So

$$\int \frac{x-2}{x^3+x} dx = \int -\frac{2}{x} dx + \int \tfrac{2x}{x^2+1} dx + \int \tfrac{1}{x^2+1} dx$$

$$= -2\ln|x| + \ln|x^2+1| + \tan^{-1}(x) + c \qquad (0.5+0.5+0.5)$$