KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1431/1432)

Solution First Mid-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 20 Time: 90 minutes

Marks:

Multiple Choice (1-10)	
Question # 11	
Question # 12	
Question # 13	
Question # 14	
Total	

Multiple Choice

Q.No:	1	2	3	4	5	6	7	8	9	10
$\{a,b,c,d\}$	d	a	c	c	c	c	b	b	b	a

Q. No: 1 Using the definition of Riemann Sum as a definite integral,

$\lim_{\|P\|\to 0} \sum_k (\omega_k)^4 \Delta x_k$, on $[0,1]$ is equal to:

(a) 0 (b) ∞ (c) 1 (d) $\frac{1}{5}$

Q. No: 2 The value of the integral $\int_0^2 x\sqrt{4-x^2}dx$ is equal to:

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{-3}{8}$ (d) $\frac{-8}{3}$

Q. No: 3 The number z that satisfies the conclusion of the Mean value Theorem for $f(x) = x$ on $[\alpha, \beta]$ (with $\alpha < \beta$ are two constants) is:

(a) α (b) $\beta + 1$ (c) $\frac{\alpha+\beta}{2}$ (d) β

Q. No: 4 The average value of $f(x) = |x-1|$ on $[0,2]$ is equal to:

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

Q. No: 5 If $F(x) = \int_1^{2x} f'(t)\,dt$, then $F'(x)$ is equal to:

(a) $2f(2x) - f(1)$ (b) $2f(2x)$ (c) $2f'(2x)$ (d) $f'(2x)$

Q. No: 6 The derivative of the function $f(x) = \sec^{-1}(e^x)$ is equal to:

(a) $\frac{1}{e^x\sqrt{e^x-1}}$ (b) $\frac{1}{\sqrt{e^x-1}}$ (c) $\frac{1}{\sqrt{e^{2x}-1}}$ (d) $\frac{1}{\sqrt{e^{2x}+1}}$

Q. No: 7 The value of the integral $\int_0^1 4^x dx$ is equal to:

(a) $\frac{4}{\ln 4}$ (b) $\frac{3}{2\ln 2}$ (c) $4\ln 4$ (d) $3\ln 4$

Q. No: 8 To evaluate $\int \frac{x^3}{\sqrt{1-x^8}}dx$, we use the change of variable:

(a) $u = x^2$ (b) $u = x^4$ (c) $u = x^8$ (d) $u = x^3$

Q. No: 9 The value of the integral $\int \frac{\sin x}{\sqrt{4-\cos^2 x}}dx$ is equal to:

$(a)\sin^{-1}(\cos x) + c$ $(b)\cos^{-1}\left(\frac{\cos x}{2}\right) + c$ $(c) - \cos^{-1}\left(\frac{\cos x}{2}\right) + c$ $(d)\sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Q. No: 10 The value of the integral $\int \frac{1}{\sqrt{4+x^2}}dx$ is equal to:

(a) $\sinh^{-1}\left(\frac{x}{2}\right)+c$ (b) $\sin^{-1}\left(\frac{x}{2}\right)+c$ (c) $\frac{1}{2}\sinh^{-1}\left(\frac{x}{2}\right)+c$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)+c$

Full Questions

Question No: 11 Approximate the integral $\int_0^4 \sqrt{1+x^3}dx$ using the **Simpson's rule** with $n=4$. [**3**]

Solution:

Let $f(x)=\sqrt{1+x^3}$.

$\Delta x=\frac{4}{4}=1$

$x_0=0,\ x_1=1,\ x_2=2,\ x_3=3$ and $x_4=4$. (1)

$$\int_0^4 \sqrt{1+x^3}dx \approx \frac{4-0}{3\times 4}\{f(0)+4f(1)+2f(2)+4f(3)+f(4)\} \quad (1)$$

$$=\frac{1}{3}\{1+4(1.4142)+2(3)+4(5.2915)+8.0623\}$$

$$=\frac{1}{3}\{1+5.6568+6+21.166+8.0623\}$$

$$=\frac{1}{3}\{41.885\}\approx 13.962 \quad (1)$$

Question No: 12 If $y=(1+x^2)^{\sin x}$, then find y'. [**2**]

Solution:

$\ln y=\sin x\ln(1+x^2)$ (1)

$$\frac{y'}{y}=\cos x\ln(1+x^2)+\frac{2x\sin x}{1+x^2}$$

So $y'=\left[\cos x\ln(1+x^2)+\dfrac{2x\sin x}{1+x^2}\right](1+x^2)^{\sin x}$ (1).

Question No: 13 Evaluate the integral $\int \frac{x+1}{\sqrt{4-x^2}}dx$. [**2**]

Solution:

$$\begin{aligned}\int \frac{x+1}{\sqrt{4-x^2}}dx &= \int \frac{x}{\sqrt{4-x^2}}dx + \int \frac{1}{\sqrt{4-x^2}}dx \\ &= -\sqrt{4-x^2} + \sin^{-1}\left(\tfrac{x}{2}\right) + c \qquad (1+1)\end{aligned}$$

Question No: 14 Evaluate the integral $\int \frac{1}{x\sqrt{1-x^4}}dx$. [**3**]

Solution:

Let $u = x^2 \Rightarrow du = 2xdx$ (1)

$$\begin{aligned}\int \frac{1}{x\sqrt{1-x^4}}dx &= \tfrac{1}{2}\int \frac{1}{u\sqrt{1-u^2}}du \qquad (1) \\ &= -\tfrac{1}{2}sech^{-1}(u) + c \\ &= -\tfrac{1}{2}sech^{-1}(x^2) + c \qquad (1)\end{aligned}$$