



KING SAUD UNIVERSITY
College of Science
Department of Mathematics

M-106

First Semester (1430/1431)

Correction Final-Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: **Three hours**

Multiple Choice(1-20)	
Question # 21	
Question # 22	
Question # 23	
Question # 24	
Question # 25	
Question # 26	
Total	

Q.No:11 The integral $\int \frac{1}{\sqrt{8-2x-x^2}} dx$, with suitable substitution is equal to:

- (a) $\int \frac{1}{\sqrt{u^2-7}} du$ (b) $\int \frac{1}{\sqrt{u^2-9}} du$ (c) $\int \frac{1}{\sqrt{9-u^2}} du$ (d) $\int \frac{1}{\sqrt{7-u^2}} du$

Q.No:12 The improper integral $\int_2^{\infty} \frac{1}{(x-1)^2} dx$,

- (a) converges to -1 (b) converges to 0 (c) converges to 1 (d) diverges

Q.No:13 The area of the region bounded by the graphs of $y = \frac{2}{x}$, $x = 1$, $x = 3$ and $y = 0$ is equal to:

- (a) $\ln 3$ (b) $2\ln(3) - 2$ (c) $2\ln(3)$ (d) $\ln(3) - 2$

Q.No:14 The arc length of the graph of the curve $y = \cosh x$, $0 \leq x \leq 4$ is equal to:

- (a) $\sinh(4) - 1$ (b) $\cosh(4) - 1$ (c) $\sinh(4)$ (d) $\cosh(4)$

Q.No:15 The surface area generated by revolving the curve of the function $f(x) = \sqrt{4-x^2}$, $-2 \leq x \leq 2$ around the **x-axis** is equal to:

- (a) 16π (b) 4π (c) 8π (d) 6π

Q.No:16 If (x, y) -coordinates of a point are $(0, -2)$ then its (r, θ) -coordinates are:

- (a) $(2, \pi)$ (b) $(2, -\pi)$ (c) $\left(2, \frac{3\pi}{2}\right)$ (d) $\left(2, \frac{-3\pi}{2}\right)$

Q.No:17 The length of the curve $r = 2 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is equal to:

- (a) $\frac{\pi}{2}$ (b) π (c) 2π (d) $\frac{2\pi}{3}$

Q.No:18 The polar equation that has the same graph as the Cartesian equation

$$x^2 + y^2 - x = 2\sqrt{x^2 + y^2} \text{ is :}$$

- (a) $r = 2 + \sin \theta$ (b) $r = 2 + \cos \theta$ (c) $r = 2 - \cos \theta$ (d) $r = 2 - \sin \theta$

Q.No:19 The polar equation $r = 2 + 2 \sin \theta$ represents:

- (a) Cardioid (b) Circle (c) Ellipse (d) Straight line

Q.No:20 The slope of the tangent line to the curve $x = 4t + 1$, $y = t^2 - 2$ at $t = 1$ is equal to:

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{4}$

Full Questions

Question No: 21 Approximate $\int_0^{\sqrt{\pi}} \sin(4x^2) dx$, using **Simpson's rule** with $n=4$. [6]

Solution: Let $f(x) = \sin(4x^2)$

$$x_0 = 0, \quad x_1 = \frac{\sqrt{\pi}}{4}, \quad x_2 = \frac{\sqrt{\pi}}{2}, \quad x_3 = \frac{3\sqrt{\pi}}{4} \quad \text{and} \quad x_4 = \sqrt{\pi} \quad (1)$$

$$\int_0^{\sqrt{\pi}} \sin(4x^2) dx \approx \frac{\sqrt{\pi}}{3 \times 4} \{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\} \quad (2)$$

$$\approx \frac{\sqrt{\pi}}{12} \{(0) + 4 \times (0.70711) + 2 \times (0) + 4 \times (0.70711) + (0)\}$$

$$\approx \frac{\sqrt{\pi}}{12} \{(0) + 2.8284 + 2 \times (0) + 2.8284 + (0)\} \quad (2)$$

$$\approx \frac{\sqrt{\pi}}{12} \{5.6568\}$$

$$\approx 0.83553$$

(1)

Question No: 22 Evaluate the integral $\int \sin^{-1}(x) dx$. [4]

Solution: By Integrations by parts (with $u = \sin^{-1} x$ and $dv = 1$) we obtain:

$$\int \sin^{-1}(x) dx = x \sin^{-1}(x) - \int \frac{x}{\sqrt{1-x^2}} dx \quad (2)$$

$$= x \sin^{-1}(x) + \sqrt{1-x^2} + c \quad (2)$$

Question No: 23 Evaluate the integral $\int \frac{2x+3}{x^2+2x+3} dx$ [5]

Solution:

$$\int \frac{2x+3}{x^2+2x+3} dx = \int \frac{2x+2}{x^2+2x+3} dx + \int \frac{1}{x^2+2x+3} dx \quad (2)$$

$$= \int \frac{2x+2}{x^2+2x+3} dx + \int \frac{1}{(x+1)^2+2} dx \quad (1)$$

$$= \ln|x^2+2x+3| + \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + c \quad (2)$$

Question No: 24 Determine whether the integral $\int_0^{\infty} \frac{e^x}{1+e^{2x}} dx$ converges or diverges [5]

Solution:

$$\int_0^{\infty} \frac{e^x}{1+e^{2x}} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{e^x}{1+e^{2x}} dx \quad (1)$$

Let $u = e^x$, then

$$\int_0^t \frac{e^x}{1+e^{2x}} dx = \int_1^{e^t} \frac{1}{1+u^2} du = \tan^{-1}(e^t) - \tan^{-1}(1) = \tan^{-1}(e^t) - \frac{\pi}{4}. \quad (2)$$

Hence

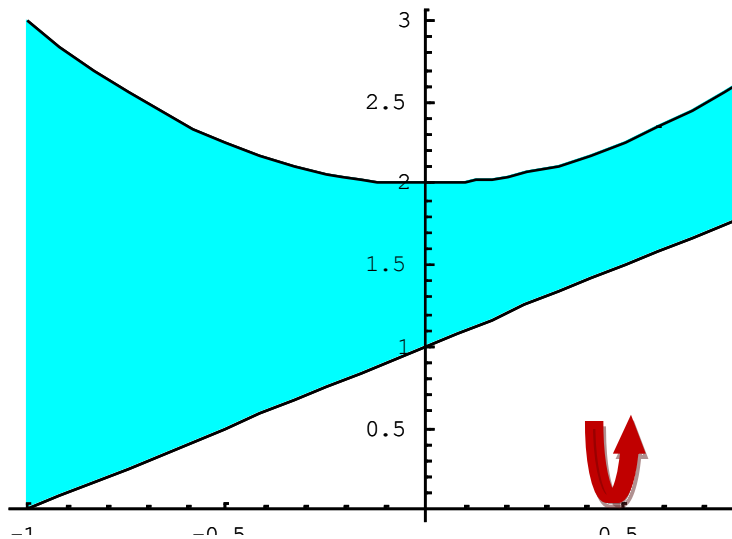
$$\int_0^{\infty} \frac{e^x}{1+e^{2x}} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{e^x}{1+e^{2x}} dx = \lim_{t \rightarrow \infty} \left[\tan^{-1}(e^t) - \frac{\pi}{4} \right] = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

That is

$$\int_0^{\infty} \frac{e^x}{1+e^{2x}} dx \text{ converges and } \int_0^{\infty} \frac{e^x}{1+e^{2x}} dx = \frac{\pi}{4}. \quad (2)$$

Question No: 25 Let R be the region bounded by the graphs of the functions $y = x^2 + 2$, $y = x + 1$, $x = -1$ and $x = 1$: Sketch the region R and set up an integral to find the volume of the solid generated by revolving the region R around the x -axis. (Use Washer Method) [5]

Solution:



(2)

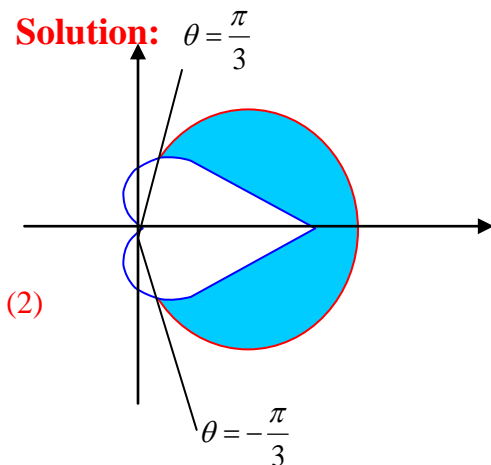
$$V = \pi \int_{-1}^1 [(x^2 + 2)^2 - (x + 1)^2] dx \quad (3)$$

Question No: 26 Let R be the region which is **outside** the graph of $r = 2 + 2 \cos \theta$ and **inside** the graph of the polar equation $r = 6 \cos \theta$:

Sketch the region R and set up an integral that can be used to find the **area** of the region R .

[5]

Solution:



(2)

$$\text{If } r = 2 + 2 \cos \theta = 6 \cos \theta \Rightarrow \theta = \pm \frac{\pi}{3} \quad (1)$$

$$A = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [(6 \cos \theta)^2 - (2 + 2 \cos \theta)^2] d\theta \quad (2)$$