King Saud University

Department of Mathematics

1st Semester 1445H

MATH 240 (Linear Algebra)

1st Midterm Exam

Duration:90 Minutes

[redacted]

Name: [redacted] 15

Question Number	I	II	III +1V	Total
Mark	2.5	3	3.5 + 3.5	12.5

Question Number	1	2	3	4	5	6	Total
Answer	a	a	b	d	c	b	

Question I:

Choose the correct answer, then fill in the table above:

(1) If A *is* 2×2 *matrix and* $\boxed{detA = 9}$ **then** $\boxed{detA^T =}$

(a) 9 **(b) -9** **(c)** $\frac{1}{9}$ **(d) None of the previous**

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(2) If A **is a** $\boxed{4 \times 5}$ **matrix and** B **is a** $\boxed{5 \times 7}$ **matrix then** $B(AB)^T A$ **is** 4 x 5

5x7 4x7

(a) 5 × 5matrix **(b) 4 × 7 matrix** **(c)7 × 7 matrix** **(d) None of the previous**

من اليسار, اليمين

5 x 4 4 x 5 7x4

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(3) If A *and* B *are* 4×4 **matrices such that** $\boxed{tr(A) = 6}$ *and* $\boxed{tr(5A^T - I) = trB}$**, then** $\boxed{trB =}$

→ 5(6) −

30 − 4

(a) 23 **(b) 26** **(c) 28** **(d) None of the previous**

I 4

(4) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$**, then** A **is**

(a) invertible **(b) elementary** **(c) non- invertible** **(d) None of the previous**

عكسي

عرفنا ما العكس

(5) If $A = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ is the augmented matrix of a linear system in the unknowns x, y, and z, then the system has

(a) no solution (b) a unique solution (c) infinitely many solutions (d) None of the previous

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(6) If $B = \begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix}$ then $B^{-1} =$ $10 - 6 = 4$ $B^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -6 \\ -1 & 5 \end{bmatrix}$

(a) $\begin{bmatrix} \frac{1}{5} & \frac{1}{6} \\ 1 & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{4} & \frac{5}{4} \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -6 \\ -1 & 5 \end{bmatrix}$ (d) None of the previous

$\frac{1}{2}$ $-\frac{3}{2}$
$-\frac{1}{4}$ $\frac{5}{4}$

Question II:

In parts (a)–(h) determine whether the statement is true or false, and justify your answer.

(a) A linear system whose equations are all homogeneous must be consistent.

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متسقة

$0 = 0$

عدد = عدد ؟؟

(b) The system

$$x - \boxed{x}y = 4$$
$$2x - 2y = 8$$

فيها متغيرين

is a linear system.

F

REF

(c) All leading 1's in a matrix in row echelon form must occur in different columns.

موجود leading

(T)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

REF leading تحت صفر

RREF → لازم الكل فوق وتحت leading صفر

$M \times 4$ $\qquad$ 4×6

(d) If A is a 6×4 matrix and B is an $m \times n$ matrix such that BA^T is a defined matrix, then n = 4.

T

$m \times 4 \quad 4 \times 6$

$m \times 6$

(e) For all square matrices A and B of the same size, it is true that AB=BA

F

$AB \neq BA$

لأنه ناتج مختلفة

لو ضربت مصفوفتين

ترتيب بالعكس

غير قابل للعكس

(f) If A is an $n \times n$ matrix that is not invertible, then the linear system $Ax = 0$ has infinitely many solutions.

(T)

$= 0$?

(g) A symmetric matrix must be a square matrix.

T

2×2

3×3 ??

المعكوس

مصفوفة مثلثة

لنفس النوع

(h) The inverse of a triangular matrix is a triangular matrix of the same kind.

A^{-1}

A

F

uppar $\qquad$ عندي

lower

يختلف نوعها

x y z w , 47

Question III:

A. Use Cramer's rule to solve the linear system

$$\begin{cases} x+y+z=1 \\ 3x+2y-2z=-1 \\ 4x+3y-2z=0 \end{cases}$$

$x =$ 1 ✗	$y =$ 0 ✗	$z =$ 18 ✗

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 2 & -2 & -1 \\ 4 & 3 & -2 & 0 \end{array}\right]$$

$x = \dfrac{\det A_1}{\det |A|}$

$y = \dfrac{\det A_2}{\det |A|}$

$z = \dfrac{\det A_3}{\det |A|}$

$-3R_1 + R_2$
$-4R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & -1 & -6 \end{bmatrix}$$

$\det|A| = (6-5) = 1$ ✓

$$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -2 \\ 0 & 3 & -2 \end{bmatrix}$$

$1(-4-(-6)) - 1(-2) + 1(-3)$

$2 + 2 - 3 = 1$

$x = \frac{1}{1} = 1$

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & -2 \\ 4 & 0 & -2 \end{bmatrix}$$

$1(2)(-2) - 1(-6-(-8)) + 1(-4)$

$2 - 2 - 4 = -4 - 4 = 0$

$y = \frac{0}{1} = 0$

$$A_3 = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 4 & 3 & 0 \end{bmatrix}$$

1.5

$+1(-3) - 1(-4) + 1(9+8)$ 17

$-3 + 4 + 17 = \frac{18}{1}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 4 & 9 \\ 0 & 0 & 0 & 0 & -9 \end{array}\right]$$

$\rightarrow$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x + z + w = 4$

$w = 4 - x - z$

$y = -11 - (\quad)$

$4 - x - z$

B. Solve the linear system

$y - w = -11$

$$\begin{cases} x + y + z + w = 4 \\ 3x + 2y - 2z - w = 3 \\ x + 3y - z = -2 \end{cases}$$

$x = t$	$y = -11 - 4 - t - r$	$z = r$	$w = 4 - t - r$

Let $x = t$

$z = r$

$x, z \in R$

$$\begin{array}{c} \begin{array}{ccccc} x & y & z & w & -12 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 3 & 2 & -2 & -1 & 3 \\ 1 & 3 & -1 & 0 & -2 \end{array}\right] \end{array}$$

$-3R_1 + R_2$

$-R_1 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -5 & -4 & -9 \\ 0 & 2 & -2 & -1 & -6 \end{array}\right]$$

$R_2 + R_1$

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$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & -3 & -5 \\ 0 & -1 & -5 & -4 & -9 \\ 0 & 2 & -2 & -1 & -6 \end{array}\right]$$

$2R_2 + R_3$

$-10 \quad -8 \quad -18$

$-2 \quad -1 \quad -6$

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & -3 & -5 \\ 0 & -1 & -5 & -4 & -9 \\ 0 & 0 & -12 & -9 & -24 \end{array}\right]$$

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$~~-3R_1 + R_3~~$

$-R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & -3 & -5 \\ 0 & 1 & 5 & 4 & 9 \\ 0 & 0 & 0 & 0 & -9 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 4 & 9 \\ 0 & 0 & 0 & 0 & -9 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{cccc} & y & & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & -1 & -11 \\ 0 & 0 & 0 & 0 & -9 \end{array}\right] \end{array}$$

RREF

Question IV:

A. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & -1 & 1 \\ 2 & 1 & -4 \end{bmatrix}$ then find A^{-1}.

$[A^{-1} \mid I] \sim [I \mid A^{-1}]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 \\ 2 & 1 & -4 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 + R_2 \\ -2R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ \boxed{0 \quad 0 \quad -1} & & & 1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array}\right] \quad -2R_2 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array}\right] \quad R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} -R_2 \\ -R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 \end{array}\right] \leftarrow [I \mid A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -3 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

6

X

1/2

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$0+0)),(0+0+0)\ (0+0+0)$

$+0\ 0)\ (0+4+0)\ (0+0+0)$

$0+0+0)\ (0+0+0)\ (0+0+9)$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{matrix}$$

B. For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\boxed{\textbf{Find } p(A),}$ **where $p(x) = x^4 - 3x^2 - 2$.**

$$(A^2)^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$P(A) = A^4 - 3A^2 - 2$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} - 2 \ ?$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 81 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -27 \end{bmatrix} - 2 \ ?$$

$$\begin{array}{r} 27 \\ 54 \\ \hline 81 \end{array}$$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 54 \end{bmatrix} - 2 \ ?$$

1.5

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 52 \end{bmatrix}$$