King Saud University

College of Sciences

Department of Mathematics

151 Math Exercises

(1)

Propositional Logic

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Algebraic Properties of Connectives

بفرض p , q , r تقارير (Propositions) :

1) قاعدتا الإبدال (*Commutative Rules*) :

(أ) $p \wedge q \equiv q \wedge p$

(ب) $p \vee q \equiv q \vee p$

(2) قاعدتا التجميع (*Associative Rules*) :

(أ) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(ب) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(3) قاعدتا التوزيع (*Distributive Rules*) :

(أ) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(ب) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(4) قاعدتا العنصر المحايد (*Identity Rules*) :

(أ) $p \wedge T \equiv P$

(ب) $p \vee F \equiv p$

(5) قاعدتا النفي (*Negation Rules*) :

(أ) $p \vee \neg p \equiv T$

(ب) $p \wedge \neg p \equiv F$

(6) قاعدة نفي النفي (*Double Negation Rule*) : $\neg(\neg p) \equiv p$

(7) قاعدتا الجمود (*Idempotent Rules*) :

(أ) $p \vee p \equiv p$

(ب) $p \wedge p \equiv p$

(8) قاعدتا ديمورجان (*DeMorgan's Rules*) :

(أ) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(ب) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

(9) قاعدتا الشمول (*Universal Rules*) :

(أ) $p \vee T \equiv T$

(ب) $p \wedge F \equiv F$

(10) قاعدتا الإمتصاص (*Absorption Rules*) :

(أ) $p \vee (p \wedge q) \equiv p$

(ب) $p \wedge (p \vee q) \equiv p$

(11) قاعدتا البرهان البديل (*Alternative proof Rules*) :

(أ) $p \rightarrow (q \vee r) \equiv (p \wedge \neg q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow q$

(ب) $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

(12) قاعدتا الشرط (*Conditional Rules*) :

(أ) $p \rightarrow q \equiv \neg p \vee q$

(ب) $\neg(p \rightarrow q) \equiv p \wedge \neg q$

(13) قواعد ثنائي الشرط (*Biconditional Rules*) :

(أ) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

(ب) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

(ج) $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$

(14) قاعدة المكافئ العكسي (*Rule of Contrapositive*) : $p \rightarrow q \equiv \neg q \rightarrow \neg p$

(15) قاعدة الإنطلاق والوصول (*Exportation – Importation Rule*):

$$p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

في التقرير الشرطي $p \to q$ ، يسمى التقرير p (المقدمة *Antecedent*)، بينما يسمى التقرير q (النتيجة *Consequent*).

يقترن بالتقرير الشرطي $p \to q$ تقارير شرطية أخرى هي :

العكس (*Converse*) : $q \to p$

المعكوس (*Inverse*) : $\neg p \to \neg q$

المكافئ العكسي (*Contrapositive*) : $\neg q \to \neg p$

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

$p \wedge q$ يكون صائباً T إذا كان كل منهما صائباً T ، عدا ذلك يكون خاطئاً .

$p \vee q$ يكون خاطئاً F إذا كان كل منهما خاطئاً F ، عدا ذلك يكون صواباً .

$p \to q$ يكون خاطئاً F إذا كان p صائباً T و كان q خاطئاً F ، عدا ذلك يكون صائباً.

$p \leftrightarrow q$ يكون صائباً T إذا كان كل منهما صائباً T ، أو إذا كان كل منهما خاطئاً F ، عدا ذلك يكون خاطئاً .

DEFINITION 1: A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a *tautology*. A compound proposition that is always false is called a *contradiction*. A compound proposition that is neither a tautology nor a contradiction is called a *contingency*.

DEFINITION 2: The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

TABLE 1 Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

TABLE 2 De Morgan's Laws.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

EXAMPLE 2 Show that $\neg(p \lor q)$ and $\neg p \land \neg q$ are logically equivalent.

TABLE 3 Truth Tables for $\neg(p \lor q)$ and $\neg p \land \neg q$.

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

EXAMPLE 3 Show that $p \to q$ and $\neg p \lor q$ are logically equivalent.

TABLE 4 Truth Tables for $\neg p \lor q$ and $p \to q$.

p	q	$\neg p$	$\neg p \lor q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

EXAMPLE 4 Show that $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ are logically equivalent.
This is the *distributive law* of disjunction over conjunction.

TABLE 5 A Demonstration That $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ Are Logically Equivalent.

p	q	r	$q \land r$	$p \lor (q \land r)$	$p \lor q$	$p \lor r$	$(p \lor q) \land (p \lor r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Exercises

Q1– Decide whether the following propositions are tautology or a contradiction or a contingency :

1) $(p \wedge q) \to (\neg p \to q)$

Solution:

p	q	$\neg p$	$p \wedge q$	$\neg p \to q$	$(p \wedge q) \to (\neg p \to q)$
T	T				
T	F				
F	T				
F	F				

(By rules " without using the truth tables")

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2) $[\neg p \wedge (p \vee q)] \to q$

Solution:

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T				
T	F				
F	T				
F	F				

(By rules " without using the truth tables)

$$[\neg p \wedge (p \vee q)] \rightarrow q \equiv \neg[\neg p \wedge (p \vee q)] \vee q \quad \text{(Conditional Rule)}$$
$$\equiv p \vee \neg(p \vee q) \vee q \quad \text{(DeMorgan's Rule)}$$
$$\equiv (p \vee q) \vee \neg(p \vee q) \quad \text{(Commutative and Associative Rules)}$$
$$\equiv T \quad \text{(Negation Rule)}$$

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3) $\neg(p \rightarrow q) \rightarrow \neg q$

Solution:

p	q	$\neg q$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T				
T	F				
F	T				
F	F				

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4) $[p \wedge (p \rightarrow q)] \rightarrow q$

Solution:

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T			
T	F			
F	T			
F	F			

5) $(p \wedge q) \rightarrow (p \rightarrow q)$

Solution:

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T			
T	F			
F	T			
F	F			

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6) $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

7) $p \wedge \neg[q \rightarrow (p \vee r)]$

p	q	r	$p \vee r$	$q \rightarrow (p \vee r)$	$\neg[q \rightarrow (p \vee r)]$	$p \wedge \neg[q \rightarrow (p \vee r)]$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

(By rules " without using the truth tables)

8) $\neg u \rightarrow [\,(u \wedge v) \rightarrow w\,]$

u	v	w	$\neg u$	$u \wedge v$	$(u \wedge v) \rightarrow w$	$\neg u \rightarrow [\,(u \wedge v) \rightarrow w\,]$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

(By rules " without using the truth tables)

9) Decide whether the following propositions are tautology or a contradiction?

$$[(p \to q) \vee (q \to r)] \to (p \to \neg r)$$

Solution:

$[(p \to q) \vee (q \to r)] \to (p \to \neg r) \equiv$

$\equiv \neg[(p \to q) \vee (q \to r)] \vee (p \to \neg r)$

$\equiv \neg[(\neg p \vee q) \vee (\neg q \vee r)] \vee (p \to \neg r)$

$\equiv \neg[(q \vee \neg q) \vee (\neg p \vee r)] \vee (p \to \neg r) \equiv \neg[T \vee (\neg p \vee r)] \vee (p \to \neg r)$

$\equiv \neg[T] \vee (p \to \neg r) \equiv F \vee (p \to \neg r) \equiv (p \to \neg r) \equiv T \text{ or } F$

$\therefore$ a contingincy where $\boxed{T \to F \equiv F, \quad T \to T \equiv T}$

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10) Show that the following proposition is a tautology :

$$[(p \vee q) \wedge \neg p] \to q$$

11) Show that the following proposition is a tautology : $(p \wedge q) \rightarrow (r \rightarrow q)$

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12) Prove the following proposition a tautology , (Don't use the truth table) :

$$(p \wedge q) \rightarrow [(q \vee r) \rightarrow p]$$

Proof:

$$(p \wedge q) \rightarrow [(q \vee r) \rightarrow p] \equiv \neg(p \wedge q) \vee [\neg(q \vee r) \vee p]$$

$$\equiv (\neg p \vee \neg q) \vee [(\neg q \wedge \neg r) \vee p]$$

$$\equiv (\neg p \vee \neg q) \vee [(\neg q \vee p) \wedge (\neg r \vee p)]$$

$$\equiv [(\neg p \vee \neg q) \vee (\neg q \vee p)] \wedge [(\neg p \vee \neg q) \vee (\neg r \vee p)]$$

$$\equiv [\neg p \vee \neg q \vee \neg q \vee p] \wedge [\neg p \vee \neg q \vee \neg r \vee p]$$

$$\equiv [(\neg p \vee p) \vee \neg q] \wedge [(\neg p \vee p) \vee (\neg q \vee \neg r)]$$

$$\equiv [T \vee \neg q] \wedge [T \vee (\neg q \vee \neg r)]$$

$$\equiv T \wedge T \equiv T$$

13) Decide whether the following proposition is a tautology

$$(p \wedge q) \to [r \to (p \vee q)]$$ (Don't use the truth tables)

Solution:

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14) Show that the following proposition is a tautology :

$$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$$

Solution:

15) Show that the following proposition is a tautology :

$$[(p \to q) \land (q \to r)] \to (p \to r)$$

Solution:

$[(p \to q) \land (q \to r)] \to (p \to r) \equiv$

$\equiv \neg[(p \to q) \land (q \to r)] \lor (p \to r)$ *(Conditional Rule)*

$\equiv \neg[(\neg p \lor q) \land (\neg q \lor r)] \lor (\neg p \lor r)$ *(Conditional Rule)*

$\equiv \neg(\neg p \lor q) \lor \neg(\neg q \lor r) \lor (\neg p \lor r)$ *(DeMorgan's Rule)*

$\equiv (p \land \neg q) \lor (q \land \neg r) \lor (\neg p \lor r)$ *(DeMorgan's Rule)*

$\equiv (p \land \neg q) \lor [(q \land \neg r) \lor (\neg p \lor r)]$ *(Associative Rule)*

$\equiv (p \land \neg q) \lor [(q \lor \neg p \lor r) \land (\neg r \lor \neg p \lor r)]$ *(Distributive Rule)*

$\equiv (p \land \neg q) \lor [(q \lor \neg p \lor r) \land (T \lor \neg p)$ *(Negation Rule)*

$\equiv (p \land \neg q) \lor (q \lor \neg p \lor r) \land T$ *(Universal Rule)*

$\equiv (p \land \neg q) \lor (q \lor \neg p \lor r)$ *(Identity Rule)*

$\equiv [\neg(\neg p \lor q) \lor (\neg p \lor q)] \lor r \equiv T \lor r \equiv T$ *(DeMorgan's & Associative& Negation& Universal Rules)*

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16) Show that the following proposition is a tautology :

$$[p \leftrightarrow (q \lor r)] \to [(\neg q \land \neg r) \lor p]$$

Solution:

17) Show that the following proposition is a contradiction :

$$[\neg (p \to q)] \wedge [q \wedge \neg r)]$$

Solution:

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18) Show that the following proposition is a contradiction :

$$[(p \to q) \vee (q \to r)] \to (p \to \neg r)$$

Solution:

Q2 : 1) Show that $\neg(p \to q)$ and $p \wedge \neg q$ *are logically equivalent*

Solution:

p	q	$\neg q$	$p \to q$	$\neg(p \to q)$	$p \wedge \neg q$
T	T				
T	F				
F	T				
F	F				

$\neg(p \to q) \equiv \neg(\neg p \vee q)$ by conditional law

$\equiv \neg(\neg p) \wedge \neg q$ by the second De Morgan law

$\equiv p \wedge \neg q$ by the double negation law

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2) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

Solution:

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$	$\neg(p \vee (\neg p \wedge q))$	$\neg p \wedge \neg q$
T	T						
T	F						
F	T						
F	F						

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ by the second De Morgan law

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$ by the first De Morgan law

$\equiv \neg p \wedge (p \vee \neg q)$ by the double negation law

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ by the second distributive law

$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$ because $\neg p \wedge p \equiv \mathbf{F}$

$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$ by the commutative law for disjunction

$\equiv \neg p \wedge \neg q$ by the identity law for **F**

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3) Show that $(p \rightarrow q) \rightarrow q \equiv (p \vee q)$

Solution:

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4) Show that $(p \rightarrow q) \rightarrow (\neg p \rightarrow r) \equiv p \vee r$

Solution:

5) Show that

$$(p \to q) \wedge (p \to r) \equiv (\neg q \vee \neg r) \to \neg p$$

Solution:

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6) Show that

$$(p \to q) \vee r \equiv (p \to q) \vee (p \to r)$$

Solution:

$(p \to q) \vee (p \to r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$ (*Conditional Rule*)

$\equiv \neg p \vee q \vee \neg p \vee r$

$\equiv [(\neg p \vee \neg p) \vee q] \vee r$ (*Commutative and Associative Rules*)

$\equiv (\neg p \vee q) \vee r$ (*Idempotent Rule*)

$\equiv (p \to q) \vee r$ (*Conditional Rule*)

7) Show that

$$(\neg p \vee \neg r) \rightarrow (p \wedge q) \equiv p \wedge (q \vee r)$$

Solution:

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8) Show that

Solution:

$$(\boldsymbol{p} \vee \boldsymbol{q}) \rightarrow \boldsymbol{r} \equiv \neg(p \vee q) \vee r \qquad \textit{(Conditional Rule)}$$

$$\equiv (\neg p \wedge \neg q) \vee r \qquad \textit{(DeMorgan's Rule)}$$

$$\equiv (\neg p \vee r) \wedge (\neg q \vee r) \qquad \textit{(Distributive Rule)}$$

$$\equiv (\boldsymbol{p} \rightarrow \boldsymbol{r}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{r}) \qquad \textit{(Conditional Rule)}$$

9) Show that

$$(p \vee q) \rightarrow (\neg p \wedge r) \equiv \neg p \wedge (q \rightarrow r)$$

Solution:

$$(p \vee q) \rightarrow (\neg p \wedge r) \equiv \neg(p \vee q) \vee (\neg p \wedge r) \quad \text{(Conditional Rule)}$$

$$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge r) \quad \text{(DeMorgan's Rule)}$$

$$\equiv \neg p \wedge (\neg q \vee r) \quad \text{(Distributive Rule)}$$

$$\equiv \neg p \wedge (q \rightarrow r) \quad \text{(Conditional Rule)}$$

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10) Show that

$$(\neg p \vee \neg r) \rightarrow (p \wedge q) \equiv p \wedge (q \vee r)$$

Solution:

11) Show that

$$(p \rightarrow q) \wedge (q \vee \neg r) \equiv (p \vee r) \rightarrow q$$

Solution:

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12) Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent ?

Solution:

13) Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent ?

Solution:

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14) Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent ?

Solution:

15) Show that $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent ?

Solution:

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16) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent ?

Solution:

17) Show that $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are logically equivalent

Solution: (By the truth table)

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T				
T	F				
F	T				
F	F				

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18) Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent ?

Solution:

19) Show that $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

Solution:

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20) Show that $(p \wedge \neg r) \rightarrow q \equiv (p \wedge \neg q) \rightarrow r$

Solution:

21) Show that $\neg q \vee \neg[\neg p \vee (p \wedge q)] \equiv \neg q$

Solution:

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22) Show that $\neg[p \wedge (q \vee r)] \equiv (p \rightarrow \neg q) \wedge (p \rightarrow \neg r)$

Solution:

23) Show that $(p \to q) \wedge [\neg q \wedge (\neg q \vee r)] \equiv \neg (q \vee p)$

Solution:

$(p \to q) \wedge [\neg q \wedge (\neg q \vee r)] \equiv (p \to q) \wedge \neg q$ *(Absorption Rule)*

$\equiv (\neg p \vee q) \wedge \neg q$ *(Conditional Rule)*

$\equiv (\neg p \wedge \neg q) \vee (q \wedge \neg q)$ *(Distributive Rule)*

$\equiv (\neg p \wedge \neg q) \vee F$ *(Negation Rule)*

$\equiv (\neg p \wedge \neg q)$ *(Identity Rule)*

$\equiv \neg (q \vee p)$ *(DeMorgan's Rule)*

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24) Show that $(p \wedge q) \to r \equiv (p \wedge \neg r) \to \neg q$

Solution:

25) Show that $[p \to (q \to p)] \land (p \to r) \land (p \to \neg r) \equiv \neg p$

Solution:

$[p \to (q \to p)] \land (p \to r) \land (p \to \neg r) \equiv$

$\equiv [p \to (q \to p)] \land [p \to (r \land \neg r)]$

$\equiv [p \to (q \to p)] \land [p \to (F)]$

$\equiv [p \to (q \to p)] \land [\neg p \lor (F)]$

$\equiv [p \to (q \to p)] \land [\neg p]$

$\equiv [\neg p \lor (q \to p)] \land \neg p$

$\equiv \neg p \land [\neg p \lor (q \to p)] \equiv \neg p$

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26) Show that $(p \land q) \to (p \to \neg q)$ and $\neg(p \land q)$ are logically equivalent .

Solution:

27) Decide whether $(p \wedge q) \to r$ is logically equivalent to $(p \to r) \wedge (q \to r)$ or not?

Solution:

p	q	r	$p \wedge q$	$p \to r$	$q \to r$	$(p \wedge q) \to r$	$(p \to r) \wedge (q \to r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

(Resolve it by rules)?

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28) Decide whether $(p \to q) \vee (p \to \neg r)$ is logically equivalent to $p \to (r \to q)$ or not?

Solution:

29) Show that

$$\neg p \vee (q \to r) \equiv (p \to r) \vee (q \to r)$$

Solution:

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30) Show that

$$p \leftrightarrow (\neg q \wedge \neg r) \equiv \neg p \leftrightarrow (q \vee r)$$ **(Don't use the truth table)**

Solution:

31) Show that $(p \wedge r) \leftrightarrow (q \wedge r)$ and $(p \leftrightarrow q) \wedge r$ are logically equivalent .

Solution:

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32) Show that the contrapositive of $(p \wedge q) \rightarrow r$ is logically equivalent to $p \rightarrow (q \rightarrow r)$

Solution:

33) Show that the contrapositive of $(p \vee q) \rightarrow r$ is logically equivalent to $\neg r \rightarrow (\neg p \vee \neg q)$

Solution:

Q3 - State the contrapositive of the following statements :

1) If mn is an odd number, then m is an odd number and also n is an odd number.

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2) If 3 divides the integers m *and* n , then 3 divides $m + n$

3) If $m . n = l$, then $m \geq 0$ or $n \geq 0$ or $l \geq 0$: $m, n, l \in \mathbb{Z}$

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4) If the integer $a + b - c$ is an even , then $a\ is\ even$ or $b\ is\ even$ or $c\ is\ even$,where $a, b, c \in \mathbb{Z}$

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5) If n is a prime number where $n \neq 2$, then n is odd .

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6) If x is integer, then x is odd or x is even .

7) If a and b are odd integers , then $a+b$ is even .

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8) If $x \geq 2$ or $y \geq 3$,then $x^2 + y^2 \geq 4$

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9) State the converse, the inverse and the contrapositive for these Propositions :

A. I will come over whenever there is a football game on .

B. I sleep until noon, whenever I stay up late the night before .

C. If it is raining, then the home team wins .

D. If you solve all exercises then you get a good mark.