EXERCISE SHEET MATH151

Section Required Exercises 1.1 2, 3, 8(a,d,g), 11(a,c,e), 17, 28, 29(a,c), 31(c,e), 35(e), 40. 1.3 1(a), 3(a), 7, 9(c), 10(c), 11, 12, 14, 16, 19, 22. 1.4 1, 5, 7, 11, 14, 15, 19. 1.7 1, 3, 6, 9, 11, 15, 16, 17, 26, 31. 1.8 1, 3, 6, 9, 14, 19, 29, 34. 5.1 4, 5, 6, 8, 9, 12, 18, 20, 28, 31, 32. Q1: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = 1$, $a_2 = 5$, $a_{n+1} = 2$ a_n 3 a_{n-1} , $for all n \ge 2. Prove that: a_n is an odd number for all n \ge 1. Q2: Let \{a_n\} be a sequence of integers defined inductively as: a_1 = a_2 = a_3 = 1, a_{n+2} = a_n + a_{n-1}, for all n \ge 2. Prove that: a_n is an odd number for all n \ge 1. Q3: Let \{a_n\} be a sequence of integers defined inductively as: a_0 = 1, a_{n+1} = a_n + 3^n, for all n \ge 0. Prove that: a_n = \frac{1}{2}(3^n + 1), for all n \ge 0. Q4: Let \{x_n\} be a sequence defined as: x_1 = 1, x_2 = 2, x_{n+2} = \frac{1}{2}(x_{n+1} + x_n), \forall n \ge 1 Prove that: 1 \le x_n \le 2. Q5: Let \{y_n\} be a sequence defined as: y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3), \forall n \ge 1. Prove that: (a)y_n < 2, for all n \ge 1. (b) y_n < y_{n+1}, for all n \ge 1. Q6: Let \{a_n\} be a sequence defined as: a_0 = 2, a_1 = 4, a_2 = 6, a_n = 5, a_{n-3}, \forall n \ge 3. Prove that: a_n is even, for all n \ge 0.$	
1.3 1(a), 3(a), 7, 9(c), 10(c), 11, 12, 14, 16, 19, 22. 1.4 1, 5, 7, 11, 14, 15, 19. 1.7 1, 3, 6, 9, 11, 15, 16, 17, 26, 31. 1.8 1, 3, 6, 9, 14, 19, 29, 34. 5.1 4, 5, 6, 8, 9, 12, 18, 20, 28, 31, 32. Q1: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = 1$, $a_2 = 5$, $a_{n+1} = 2$ a_n 3 a_{n-1} , for all $n \ge 2$. Prove that: $3^n \le a_{n+1} \le 2$ (3 ⁿ), for all $n \ge 1$ Q2: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = a_2 = a_3 = 1$, $a_{n+2} = a_n + a_{n-1}$, for all $n \ge 2$. Prove that: a_n is an odd number for all $n \ge 1$. Q3: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_0 = 1$, $a_{n+1} = a_n + 3^n$, for all $n \ge 0$. Prove that: $a_n = \frac{1}{2}(3^n + 1)$, for all $n \ge 0$. Q4: Let $\{x_n\}$ be a sequence defined as: $x_1 = 1$, $x_2 = 2$, $x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$, ∀ $n \ge 1$ Prove that: $1 \le x_n \le 2$. Q5: Let $\{y_n\}$ be a sequence defined as: $y_1 = 1$, $y_{n+1} = \frac{1}{4}(2y_n + 3)$, ∀ $n \ge 1$. Prove that: $(a)y_n < 2$, for all $n \ge 1$. (b) $y_n < y_{n+1}$, for all $n \ge 1$. Q6: Let $\{a_n\}$ be a sequence defined as: $a_0 = 2$, $a_1 = 4$, $a_2 = 6$, $a_n = 5$, a_{n-3} , ∀ $n \ge 3$. Prove that: a_n is even, for all $n \ge 0$.	
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1.7 1, 3, 6, 9, 11, 15, 16, 17, 26, 31. 1.8 1, 3, 6, 9, 14, 19, 29, 34. 5.1 4, 5, 6, 8, 9, 12, 18, 20, 28, 31, 32. Q1: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = 1$, $a_2 = 5$, $a_{n+1} = 2$ a_n 3 a_{n-1} , for all $n \ge 2$. Prove that: $3^n \le a_{n+1} \le 2$ (3^n), for all $n \ge 1$ Q2: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = a_2 = a_3 = 1$, $a_{n+2} = a_n + a_{n-1}$, for all $n \ge 2$. Prove that: a_n is an odd number for all $n \ge 1$. Q3: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_0 = 1$, $a_{n+1} = a_n + 3^n$, for all $n \ge 0$. Prove that: $a_n = \frac{1}{2}$ ($3^n + 1$), for all $n \ge 0$. Q4: Let $\{x_n\}$ be a sequence defined as: $x_1 = 1$, $x_2 = 2$, $x_{n+2} = \frac{1}{2}$ ($x_{n+1} + x_n$), ∀ $n \ge 1$ Prove that: $1 \le x_n \le 2$. Q5: Let $\{y_n\}$ be a sequence defined as: $y_1 = 1$, $y_{n+1} = \frac{1}{4}$ (2 $y_n + 3$), ∀ $n \ge 1$. Prove that: $(a)y_n < 2$, for all $n \ge 1$. (b) $y_n < y_{n+1}$, for all $n \ge 1$. (c) Let $\{a_n\}$ be a sequence defined as: $a_0 = 2$, $a_1 = 4$, $a_2 = 6$, $a_n = 5$, a_{n-3} , ∀ $n \ge 3$. Prove that: a_n is even, for all $n \ge 0$.	
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5.1 4, 5, 6, 8, 9, 12, 18, 20, 28, 31, 32. Q1: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = 1$, $a_2 = 5$, $a_{n+1} = 2$ a_n 3 a_{n-1} , $for all$ $n \geq 2$. Prove that: $3^n \leq a_{n+1} \leq 2$ (3^n) , for all $n \geq 1$ Q2: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = a_2 = a_3 = 1$, $a_{n+2} = a_n + a_{n-1}$, for all $n \geq 2$. Prove that: a_n is an odd number for all $n \geq 1$. Q3: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_0 = 1$, $a_{n+1} = a_n + 3^n$, for all $n \geq 0$. Prove that: $a_n = \frac{1}{2}$ $(3^n + 1)$, for all $n \geq 0$. Q4: Let $\{x_n\}$ be a sequence defined as: $x_1 = 1$, $x_2 = 2$, $x_{n+2} = \frac{1}{2}$ $(x_{n+1} + x_n)$, \forall $n \geq 1$ Prove that: $1 \leq x_n \leq 2$. Q5: Let $\{y_n\}$ be a sequence defined as: $y_1 = 1$, $y_{n+1} = \frac{1}{4}$ $(2$ $y_n + 3)$, \forall $n \geq 1$. Prove that: $(a)y_n < 2$, for all $n \geq 1$. $(b) y_n < y_{n+1}$, for all $n \geq 1$. Q6: Let $\{a_n\}$ be a sequence defined as: $a_0 = 2$, $a_1 = 4$, $a_2 = 6$, $a_n = 5$, a_{n-3} , \forall $n \geq 3$. Prove that: a_n is even, for all $n \geq 0$. Q7: Let $\{b_n\}$ be a sequence defined as:	
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$b_0 = 1, b_1 = 2, b_2 = 3, b_n = b_{n-1} + b_{n-2} + b_{n-3}, $ $\forall n > 3$	
Prove that: $b_n < 3^n$, for all $n \ge 1$.	
9.1 1, 3, 6, 10, 11, 18, 26, 30, 32, 34(a,d,e), 36(d,e,h), 41, 50, 51, 52, 53, 56.	
9.3 2(c,d), 3(a,b), 4(a,c), 7(a,b), 8(a,c), 13(c), 14(a,b,c), 18, 22, 24, 26, 27, 31, 32.	
9.5 1, 3, 9, 16, 21, 22, 23, 26, 28, 36, 40(a), 42, 46, 47(b), 48(a), 55, 56(a,b).	
9.6 1, 6, 9, 10, 11, 14, 20, 22.	
10.1 3, 4, 5, 6, 7, 8, 9, 10.	
10.1 3, 4, 5, 6, 7, 8, 9, 10. 10.2 1, 2, 3, 4, 5, 6, 20(a,b,c,d), 21, 22, 23, 24, 25, 26(a,b), 35, 36, 37, 38, 39, 40, 41, 53(a,b), 59, 60	
10.2 1, 2, 3, 4, 5, 6, 20(a,b,c,u), 21, 22, 23, 24, 25, 26(a,b), 35, 36, 37, 38, 39, 40, 41, 53(a,b), 59, 60 10.3 34, 35, 36, 37, 38, 39, 53, 54, 55.	•
10.5 54, 53, 56, 57, 58, 59, 53, 54, 53.	
10.4 1, 2, 3, 4, 3, 6. 11.1 2, 4, 6, 10, 16, 17.	
11.1 2, 4, 0, 10, 10, 17. 11.2 1, 2.	
11.2 1, 2. 11.3 None	
11.4 2, 3, 4, 5, 6, 7(a, b, c, e, f), 8.	
12.1 1, 2, 3(a), 4(a), 5(b,d), 6(c,d), 11, 28.	
12.2 1, 2(a, b), 3(a, b), 11(a, b).	
12.4 1, 2, 3, 4(c), 6(a,b), 12, 13, 14.	