King Saud University
College of Science
Department of Mathematics

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    Course Title: Mathematical logic
    Course Code: 132 Math
Course Instructor: Reem Almahmud
            Exam: 1 }\mp@subsup{}{}{\mathrm{ st }}\mathrm{ MIDTERM
        Semester: 1 1 term 1445/1446
            Date: 04-10-2023
        Duration: 2 Hrs
            Marks: 25
```

Privileges: Calculator is not Permitted

## Student Name:

Student ID:
Section No: 54945
Serial No:

## Instructions:

- Cell Phones should be switched off or on silent mode during the exam.
- Write your answers directly on the question sheet.
- There are 4 questions in 5 pages.

| Official Use Only |  |  |
| :---: | :---: | :---: |
| Question | Students Marks | Question Marks |
| $\mathbf{1}$ |  | 6 |
| $\mathbf{2}$ |  | 7 |
| $\mathbf{3}$ |  | 7 |
| $\mathbf{4}$ |  | 5 |
| Total |  | 25 |


| $(1)$ | $(2)$ | $(3)$ | (4) | (5) | (6) |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

Q1: Choose the correct answer and write it in the top table:

1- The truth value of the statement "For all positive integers $x$, if $x^{2}<0$, then $x+1>2$ "
a) True
b) False
c) Undetermined
d) None

2- Let $P(x, y)$ be the statement " $x-3 y-6=0$ ". Then,
a) $P(1,6)$ is true
b) $P(9,1)$ is true
c) $P(1,9)$ is true
d) $P(6,1)$ is true

3- $\neg[\forall x(1+2 x \geq 2-x \vee x>0)]$ is equivalent to
a) $\exists x(1+2 x<2-x \wedge x \leq 0)$
b) $\forall x(1+2 x<2-x \wedge x \leq 0)$
c) $\exists x(1+2 x<2-x \vee x \leq 0)$
d) $\exists x(1+2 x \geq 2-x \vee x>0]$

4- Let $Q(x)$ be the statement " $x+4>3 x$ ", with the domain tb the set of integers. Which of the following statements is true:
a) $\forall x Q(x)$
b) $\exists x Q(x)$
c) $Q(5)$
d) $Q(10)$

5- The inverse of the conditional statement "If $n$ is odd, then $n^{2}$ is odd" is
a) If $n^{2}$ is even, then $n$ is even
b) If $n^{2}$ is odd, then $n$ is odd
c) If $n$ is even, then $n^{2}$ is even
d) None

6- The proposition $(p \vee \neg p) \rightarrow p$ is a
a) Tautology
b) Contradiction
c) Contingency
d) None

Q2: I - Prove the following statement:
I- $n^{2}+1 \geq 2^{n}$ where $n$ is a positive integer with $1 \leq n \leq 4$.

II - Prove that $[\neg \mathrm{p} \wedge(\mathrm{p} \vee \mathrm{q})] \rightarrow \mathrm{q}$ is a tautology, (Use two different ways).

Q3: Prove the following statements:
I- If $n$ is integer and $3 n+2$ is even, then $n$ is even. (Use two different methods)

II - If $m+n$ and $n+p$ are even integers, where $m, n$ and $p$ are integers, then $m+p$ is even. (Use direct method)

Q4: Prove that

$$
1^{2}+2^{2}+\cdots+n^{2}=\frac{n(n+1)(2 n+1)}{n}, \text { for a positive integer } n
$$

