King Saud University

Faculty of Sciences

Department of Mathematics

Second Examination Math 132 Semester I 1439-1440
Time: 1H30

Exercise 1 : (4+3)

1. Consider the two sets $A := \{1, 2, 3, 4, \{1\}, \{2\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{2\}\}\}$ and $B := \{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$. Determine whether each of the following seven statements is true or false. (Justify your answer).
   (i) S_1: "$\{1, 2\} \in A$". **(ii)**S_2: "$\{1, 2\} \subseteq A$". **(iii)**S_3: "$\{1, \{2\}\} \subseteq A$".
   (iv) S_4: "$A \cap \{1, 2, \{\{1\}, \{2\}\}\} = \{1, 2\}$". **(v)** S_5: "$\emptyset \in B$".
   (vi) S_6: "$\{\{\emptyset\}\} \subseteq B$". **(vii)** S_7: "$(\{\emptyset, \{\emptyset\}\} \cap \{\emptyset, \{\emptyset, \{\emptyset\}\}\}) \subseteq B$".
2. Consider the following three sets $C := \{a, b, c\}$, $D := \{1, 2, a\}$, and $E := \{(a, 1), (1, a), (b, b), (2, 2), (c, 2), (c, a)\}$. Find the following sets:
   (i) $(C \cap D) \times C$. **(ii)** $E \setminus (C \times D)$. **(iii)** $\{\emptyset\} \times E$.

Exercise 2 : (2+2+(2+1+2))
Let R be the relation from the set $A := \{1, 3, 5, 7\}$ to the set $B := \{0, 1, 2, 3, 4\}$ defined as follows: for $a \in A$ and $b \in B$, $[(aRb) \Leftrightarrow (a \leq b)]$.
In this exercise, the elements of each of the sets A and B are listed in icreasing order.

1. List all the ordered pairs in the relation R.
2. Represent the relation R with a matrix.
3. Let S be the relation from B to A defined by $S := \{(0, 1), (1, 1), (2, 1), (3, 5)\}$.
   (a) Find the following relations: $S^{-1} \cap R$ and $S \circ R$.
   (b) Draw the digraph of the relation $S \circ R$.
   (c) Represent the relation $(S \circ R)^2$ with a matrix.

Exercise 3 : ((2+2+2)+(2+1))

1. Consider the relation $R := \{(a, a), (c, c), (d, d), (e, e), (f, f), (a, f), (f, a), (b, e)\}$, defined on the set $A := \{a, b, c, d, e, f\}$.
   (a) For the relation R, determine whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. (Justify your answer).
   (b) Prove that the relation E, defined on the set A by $E := R \cup \{(b, b), (e, b)\}$, is an equivalence relation on A.
   (c) Find the equivalence classes of the equivalence relation E.
2. Consider the partial ordering P of divisibility on the set $B := \{1, 2, 3, 6, 7, 8\}$ (that is, $P := \{(a, b) \in B \times B : a \text{ divides } b\}$).
   (a) Draw the Hasse diagram of P.
   (b) Is P a total ordering?