Department of Statistics and Operations Research

College of Science

King Saud University

OR 441 Second Mid-term Examination

Semester 2, 1442 H

Name of Student: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_

 Student’s Number: \_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_

Section number:\_\_\_\_\_\_\_\_\_\_\_\_

**Question 1 (5 marks)**

Develop a generation scheme for the triangular distribution with cdf (a=2, b=3, c=6)



Generate 2 values of the random variate using the following five random numbers R~U(0,1).

R1= 0*.*2357, R2=0.8353,

Solution



**Question 2 (6 marks)**

Data have been collected on service times at a drive-in bank window at the Shady Lane National

Bank. This data is summarized into intervals as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Interval (second) | frequency | probability | Cumulative probability | slope |
| 15-30 | 10 |  |  |  |
| 30-45 | 20 |  |  |  |
| 45-60 | 25 |  |  |  |
| 60-90 | 35 |  |  |  |
| 90-120 | 30 |  |  |  |
| 120-180 | 20 |  |  |  |
| 180-300 | 10 |  |  |  |

1. Fill the table above for generating service times,
2. Draw empirical continuous cdf of service times
3. Generate 2 values of the random variate using the following two random numbers R~U(0,1).

R1= 0*.*2357, R2=0.8353,

Solution

The table look-up method for service times:



**Question 3 (4 marks)**

Times to failure for an automated production process have been found to be randomly distributed

with a Weibull distribution with parameters β *=* 2 and *α* = 10. Using inverse technique derive equation for generating Weibull random numbers, and then use it to generate two values from this Weibull distribution, using the following five random numbers R~U(0,1). Hint: *F(x)=1-exp(-(x/ α)β )*

R=0.8353, 0.2004

Solution



**Question 4 (5 marks)**

Write algorithm for generating random numbers from Poisson distribution with mean α= 0*.*2, then using a sequence of random numbers R~U(0,1), generate three Poisson random numbers
 R= 0*.*4357, 0.4146, 0.8353, 0.9952, 0.8004

Solution

Compute: exp (- α) = 0*.*8187.

**Step 1.** Set *n* = 0*, P* = 1.

**Step 2.** Generate a random number *Rn*+1, and replace *P* by *P* · *Rn*+1.

**Step 3.** If *P* < exp (- α) = 0*.*8187, then accept *N* = *n*. Otherwise, reject the current *n*, increase *n* by one, and return to step 2.

The calculations required for the generation of these three Poisson random variates are summarized

as follows:



**Question 5 (10 marks)**

A milling machine has three different bearings that fail in service. The distribution of the life of each bearing is identical, shown in the following Table

|  |  |  |  |
| --- | --- | --- | --- |
| Bearing Life (Hours) | Probability | Cumulative probability | Intervals |
| 1000 | 0.10 |  |  |
| 1100 | 0.13 |  |  |
| 1200 | 0.25 |  |  |
| 1300 | 0.13 |  |  |
| 1400 | 0.09 |  |  |
| 1500 | 0.12 |  |  |
| 1600 | 0.02 |  |  |
| 1700 | 0.06 |  |  |
| 1800 | 0.05 |  |  |
| 1900 | 0.05 |  |  |

 When a bearing fails, the mill stops, a mechanic is called, and he or she installs a new bearing (costing $32 per bearing). The delay time for the mechanic to arrive varies randomly, having the distribution given in the following Table.

|  |  |  |  |
| --- | --- | --- | --- |
| Delay Time (minutes) | Probability | Cumulative probability | Intervals |
| 5 | 0.6 |  |  |
| 10 | 0.3 |  |  |
| 15 | 0.1 |  |  |

 Downtime for the mill is estimated to cost $5 per minute. The direct on-site cost of the mechanic is $15 per hour.

The mechanic takes 20 minutes to change one bearing, 30 minutes to change two bearings, and 40 minutes to change three bearings. The bearings cost $16 each.

 The engineering staff has proposed a new policy to replace all three bearings whenever one bearing fails. Management needs an evaluation of the proposal, using total cost per 10,000 bearing-hours as the measure of performance.

The table below shows simulation for the current policy

|  |  |  |  |
| --- | --- | --- | --- |
|  | Bearing 1 | Bearing 2 | Bearing 3 |
|  | R.N | Life(Hours) | Cum. Life (Hours) | R.N | Delay (minutes) | R.N | Life(Hours) | Cum. Life (Hours) | R.N | Delay (minutes) | R.N | Life(Hours) | Cum. Life (Hours) | R.N | Delay (minutes) |
| 1 | 0.67 |  |  | 0.22 |  | 0.74 |  |  | 0.01 |  | 0.76 |  |  | 0.02 |  |
| 2 | 0.08 |  |  | 0.35 |  | 0.43 |  |  | 0.74 |  | 0.65 |  |  | 0.25 |  |
| 3 | 0.49 |  |  | 0.10 |  | 0.86 |  |  | 0.31 |  | 0.61 |  |  | 0.74 |  |
| 4 | 0.84 |  |  | 0.72 |  | 0.93 |  |  | 0.12 |  | 0.96 |  |  | 0.18 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 110 |  |  |  |  | 125 |  |  |  |  | 95 |

If 46 bearing were changed and the sum of delay time for each bearing is given then find the following:

1. Cost of bearings
2. Cost of delay time
3. Cost of downtime during repair
4. Cost of repairpersons
5. Total cost







* + The cost of the current system is estimated as follows:
		- Cost of bearings = 46 bearings × $16/bearing = $736
		- Cost of delay time = (110 + 125 + 95) minutes × $5/minute = $1650
		- Cost of downtime during repair =

 46 bearings × 20 minutes/bearing × $5/minute = $4600

* + - Cost of repairpersons =

 46 bearings × 20 minutes/bearing × $15/60 minutes = $230

* + - Total cost = $736 + $1650 + $4600 + $230 = $7216