109-mid2-443-sol

1. $\hat{p}_B \sim N\left(p_B, \dfrac{p_B q_B}{n_B}\right)$

2. 0.51

3. $P(p_A < 0.5) = P\left(Z < \dfrac{0.5 - 0.43}{\sqrt{\dfrac{0.43 \times 0.57}{90}}}\right) = P(Z < 1.34) = 0.90988$

4. $\sqrt{\dfrac{p_A q_A}{n_A} + \dfrac{p_B q_B}{n_B}} = \sqrt{\dfrac{0.43 \times 0.57}{90} + \dfrac{0.51 \times 0.49}{150}} = 0.06625$

5. $P(\hat{p}_A - \hat{p}_B > 0.05) = 1 - P\left(Z < \dfrac{0.05 - (0.43 - 0.51)}{0.06625}\right)$
$$= 1 - P(Z < 1.96)$$
$$= 1 - 0.975 = 0.025$$

6. $n_A \geq 30$, $n_A p_A > 5$, $n_A q_A > 5$

$$\boxed{\lambda_{one\ month} = 10}$$

7. $P(X = 2) = \dfrac{e^{-10} 10^2}{2!} = 0.0023$

8. $P(X = 3) = \dfrac{e^{-10} 10^3}{3!} = 0.0076$

9. $\sqrt{\lambda_{one\ year}} = \sqrt{10 \times 12} = \sqrt{120} = 10.95$

$$\boxed{k = 0.25}$$

10. $P(1.5 < X < 3.5) = 0.6$

11. $P(X \leq 1) = 0.35$

12. $P(X = 2.5) = 0$

13. $E(X) = (0 \times 0.15) + (1 \times 0.25) + (2 \times 0.4) + (3 \times 0.2) = 1.65$

14. $E(X^2) = (0^2 \times 0.15) + (1^2 \times 0.25) + (2^2 \times 0.4) + (3^2 \times 0.2) = 3.65$

$$Var(X) = E(X^2) - E(X)^2 = 3.65 - (1.65)^2 = 0.9275$$

$$\boxed{p = 0.45 \ , \ n = 6}$$

15. Binomial

16. $P(X = 4) = \binom{6}{4}(0.45)^4(0.55)^2 = 0.1861$

17. $P(X \geq 2) = 1 - P(X < 2) = 1 - 0.163567 = 0.8364$

18. $E(X) = np = 6 \times 0.45 = 2.7$

19. $E(X) = npq = 6 \times 0.45 \times 0.55 = 1.485$

$$\boxed{\mu_1 = \mu_2 \quad \begin{array}{ll} \sigma_1^2 = 100 & n_1 = 25 \\ \sigma_2^2 = 80 & n_2 = 16 \end{array}}$$

20. Zero

21. $\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{100}{25} + \frac{80}{16} = 9$

22. $P(\bar{X}_1 - \bar{X}_2 < 8) = P\left(Z < \frac{8-0}{\sqrt{9}}\right) = P(Z < 2.67) = 0.9962$

23. $P(\bar{X}_1 - \bar{X}_2 > 8) = 1 - P\left(Z < \frac{3-0}{\sqrt{9}}\right)$
$$= 1 - P(Z < 1) = 1 - 0.8413 = 0.1587$$

24. n, p

25. Location

26. 1

27. $v = 5 \ , \ t_{0.99} : 3.365$

28. 165

29. $N(165, 7.5)$

30. $P(\bar{X} > 178) = 1 - P\left(Z < \frac{178-165}{7.5}\right)$
$$= 1 - P(Z < 1.73)$$
$$= 1 - 0.9582 = 0.0418$$