

Question: 1.(a)Given

$$\begin{aligned}3x - 2y + 5z &= -1 \\x - y + z &= -4 \\-2x + 2y - z &= 7\end{aligned}$$

- i. Write the system of equations in the form $AX=B$ where A, X and B are matrices,
 - ii. Use elementary row operation to find A^{-1} , and
 - iii. Use A^{-1} to solve the given system. [10]
- (b) Use properties of the determinants to evaluate

$$\begin{vmatrix} 1 & 0 & -3 & 4 & 2 \\ 6 & 3 & 4 & 9 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{vmatrix} \quad [5]$$

Question:2 (a) Use Cramer's rule to solve the system of equations

$$\begin{aligned}x + 3y + z &= 4 \\2x + 2y + z &= -1 \\2x + 3y + z &= 3\end{aligned} \quad [10]$$

(b) Let the equation of the plane P_1 be $x - 2y - z = 4$

- i. Find the equation of the line orthogonal to the plane P_1 and passing through the point $(2, -3, 6)$,
- ii. Find the equation of the plane P_2 passing through the point $(2, -1, 2)$ and parallel to plane P_1 .
- iii. Find the distance between the planes P_1 and P_2 . [10]

Question: 3 (a) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy + y^2}{x^2 + y^2}$ does not exist. [5]

(b) Let $r(t) = \cos 2t i + \sin 2t j - tk$ be the position vector of a particle at time t . Find the tangential and the normal components of acceleration at time t [10]

Question: 4 (a) If $f(x, y) = 2x^2y^2 + xy^3$ show that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 4f(x, y) \quad [10]$$

(b) Find the directional derivative of $f(x, y, z) = x^2yz$ at the point $(1, -1, 1)$ in the direction of the normal to the plane $4x - 3z = 16$. [10]

(c) Show that the surfaces $x^2 + 4y + z^2 = 0$ and $x^2 + y^2 + z^2 - 6z + 7 = 0$ have the same tangent plane at the point $(0, -1, 2)$. [10]

Question: 5 (a) Find local extrema and saddle points, if any, on the surface

$$f(x, y) = x^3 + 3x^2 - 9x + y^3 - 12y. \quad [10]$$

(b) If $f(x, y, z) = 2x^2 + 3y^2 + z^2$, use Lagrange multipliers to find the point on the plane $2x - 3y + 4z = 15$ at which $f(x, y, z)$ has its minimum value. [10]